Chain ladder correlations

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Overview

• The chain ladder produces cell-by-cell forecasts of future claims experience
• There are two distinct families of model that support the chain ladder algorithm
• Consideration will be given to the correlations between the forecasts of different cells (conditional on information to date)
  – Separately for the two families
• Paper to appear shortly as:
  “Chain ladder correlations”. Variance 5(2).
Framework and notation

• Claims reserving trapezium

Development period \( j = 1 \, 2 \, 3 \, \ldots \, J \)

Accident period \( k = 1 \, 2 \, 3 \, \ldots \, K \)

\( Y_{kj} \)

Often in the literature \( K = J \) and the trapezium becomes a triangle

Calendar time moves in this direction
Calendar periods are represented along diagonals
Framework and notation

- Claims reserving trapezium

Development period $j = 1 \ 2 \ 3 \ \ldots \ J$

Accident period $k = 1 \ 2 \ \ldots \ K$

Past

\[ Y_{kj} \]

Future

To be predicted on the basis of past

Cumulative row sums

\[ X_{kj} = \sum_{i=1}^{\min(J,K-k+1)} Y_{ki} \]

To be predicted on the basis of past
Chain ladder algorithm

• Claims reserving trapezium

Cumulative row sums

\[ X_{kj} = \sum_{i=1}^{\min(J, K-k+1)} Y_{ki} \]

Calculate

\[ \hat{f}_j = \sum_{k=1}^{K-j} X_{k,j+1} / \sum_{k=1}^{K-j} X_{kj} \]

Forecast

\[ \hat{X}_{kj} = X_{k,K-k+1} \hat{f}_{K-k+1} \ldots \hat{f}_{j+1} \]

Last diagonal from the past

Correlations of predictions
Consider correlations of within-row forecasts conditioned on most recent information

$$\text{Corr} [X_{k,j+m}X_{k,j+m+n}|X_k]$$
Correlations of predictions

Consider correlations of within-row forecasts conditioned on most recent information

$$\text{Corr} \{X_{k,j+m}, X_{k,j+m+n} | X_{k,j}\} = \rho_{k,j+m,j+m+n}$$

ODP Mack model

- (ODPM1) Accident periods are stochastically independent, i.e. $Y_{kj}$, $Y_{hi}$ are independent if $k \neq h$
- (ODPM2) For each $k$ the $X_{kj}$ ($j$ varying) form a Markov chain
- (ODPM3) For each $k=1,2,\ldots,K$ and $j=1,2,\ldots,J-1,$

$$X_{k,j+1} | X_{k,j} \sim \text{ODP}(f_j, \phi_{j+1})$$

Parameters $f_j$ are referred to as age-to-age factors

Recursive because each observation depends on predecessor in same row

Example of a recursive model
ODP cross-classified model

- \textbf{(ODPC1)} All random variables $Y_{kj}$ are stochastically independent
- \textbf{(ODPC2)} For each $k = 1, 2, \ldots, K$ and $j = 1, 2, \ldots, J$,
  a) $Y_{kj} \sim \text{ODP}(\alpha_k, \beta_j, \phi_j)$
  b) $\sum_{j=1}^{J} \beta_j = 1$

Example of a non-recursive model

Relevance of ODP Mack and cross-classified models

- Very different models
- But in both cases chain ladder algorithm gives MLE forecasts
- But what about correlations of the forecasts under the respective models?
Conditional correlations of predictors

- Explicit expressions for $\rho_{k,j+m,j+m+n|j} = \text{Corr}[X_{k,j+m}, X_{k,j+m+n}|X_{k,j}]$ can be obtained (see paper)
- These are not especially informative in themselves
- Subsequent discussion concentrates on their properties

Properties of conditional correlations

- ODP Mack model
  - $\rho_{k,j+m,j+m+n|j} \downarrow$ as $n \uparrow$
Properties of conditional correlations

• ODP Mack model

\[ \rho_{k,j+m,j+m+n|j} \downarrow \text{as } n \uparrow \]

- Correlation decreases as separation increases

\[ \rho_{k,j+m,j+m+n|j} \uparrow \text{as any } \phi_i \uparrow, i=j+1,\ldots,j+m \]
- \[ \phi_i \downarrow, i=j+m+1,\ldots,j+m+n \]
Properties of conditional correlations

• ODP Mack model

\[ \rho_{k,j+m,j+m+n|j} \downarrow \text{ as } n \uparrow \]
• \[ \rho_{k,j+m,j+m+n|j} \uparrow \text{ as any } \]
  \[ - \varphi_i \uparrow, \ i = j+1, \ldots, j+m \]
  \[ - \varphi_i \downarrow, \ i = j+m+1, \ldots, j+m+n \]

Correlation increases as over-dispersion increases in this region
...and decreases in this region

Properties of conditional correlations

• ODP Mack model

\[ \rho_{k,j+m,j+m+n|j} \downarrow \text{ as } n \uparrow \]
• \[ \rho_{k,j+m,j+m+n|j} \uparrow \text{ as any } \]
  \[ - \varphi_i \uparrow, \ i = j+1, \ldots, j+m \]
  \[ - \varphi_i \downarrow, \ i = j+m+1, \ldots, j+m+n \]
• \[ \rho_{k,j+m,j+m+n|j} \uparrow \text{ as any } f_i \uparrow, \ i = j+1, \ldots, j+m+n \text{ provided that } \]
  \[ - \varphi_i f_i / (f_i - 1) \uparrow, \ i = j+1, \ldots, j+m \]
  \[ - \varphi_i f_i / (f_i - 1) \downarrow, \ i = j+m+1, \ldots, j+m+n \]
Properties of conditional correlations

• ODP Mack model

\[ \rho_{k,j+m,j+m+n|j} \downarrow \text{as } n \uparrow \]
\[ \rho_{k,j+m,j+m+n|j} \uparrow \text{as any } \phi_i \uparrow, i=j+1, \ldots, j+m \]
\[ \phi_i \downarrow, i=j+m+1, \ldots, j+m+n \]
\[ \rho_{k,j+m,j+m+n|j} \uparrow \text{as any } f_i \uparrow, i=j+1, \ldots, j+m+n \text{ provided that} \]
\[ \phi_i f_i /(f_i -1) \uparrow, i=j+1, \ldots, j+m \]
\[ \phi_i f_i /(f_i -1) \downarrow, i=j+m+1, \ldots, j+m+n \]

Correlation increases as claims development increases in this region

...and decreases in this region

Properties of conditional correlations

• ODP Mack model

• ODP cross-classified model

\[ \rho_{k,j+m,j+m+n|j} \downarrow \text{as } n \uparrow \]
\[ \rho_{k,j+m,j+m+n|j} \uparrow \text{as any} \]
\[ \phi_i \uparrow, i=j+1, \ldots, j+m \]
\[ \phi_i \downarrow, i=j+m+1, \ldots, j+m+n \]
\[ \beta_i \uparrow, i=j+1, \ldots, j+m \]
\[ \beta_i \downarrow, i=j+m+1, \ldots, j+m+n \]

Again effect of increasing or decreasing claims development
Claims development in recursive and non-recursive models

• Claims development effects of correlations expressed in terms of:
  – The $f_i$ for recursive models
  – The $\beta_i$ for non-recursive models
• **BUT** the two can be shown to be related:
  – $f_j = \sum_{i=j}^{j+1} \beta_i / \sum_{i=j}^{j} \beta_i$

• Then...

Properties of conditional correlations (cont’d)

• **ODP Mack model**
  - $\rho_{k,j+m,j+m+n} \downarrow$ as $n \uparrow$
  - $\rho_{k,j+m,j+m+n} \uparrow$ as any
    - $\varphi_i \uparrow, i=j+1, \ldots, j+m$
    - $\varphi_i \downarrow, i=j+m+1, \ldots, j+m+n$
  - $\rho_{k,j+m,j+m+n} \uparrow$ as any $f_i$, $i=j+1, \ldots, j+m+n$ provided that
    - $\varphi_i f_i / (f_i - 1) \uparrow, i=j+1, \ldots, j+m$
    - $\varphi_i f_i / (f_i - 1) \downarrow, i=j+m+1, \ldots, j+m+n$

• **ODP cross-classified model**
  - $\rho_{k,j+m,j+m+n} \downarrow$ as $n \uparrow$
  - $\rho_{k,j+m,j+m+n} \uparrow$ as any
    - $\varphi_i \uparrow, i=j+1, \ldots, j+m$
    - $\varphi_i \downarrow, i=j+m+1, \ldots, j+m+n$
  - $\rho_{k,j+m,j+m+n} \uparrow$ as any $f_i$, $i=j+1, \ldots, j+m+n$ provided that
    - $\Phi_i f_i / (f_i - 1) \uparrow, i=j+1, \ldots, j+m$
    - $\Phi_i f_i / (f_i - 1) \downarrow, i=j+m+1, \ldots, j+m+n$

Now almost identical
Comparison between correlations of recursive and non-recursive models (1)

- Consider the case of recursive and non-recursive models with common values of the $f_i, \varphi_i$
- The models have been seen to have similar ordering properties
- Are they actually different?

Comparison between correlations of recursive and non-recursive models (2)

- Denote $\rho_{k,j+m,j+m+n|j}$ by $\rho_{k,j+m,j+m+n|j}^{NR}$ for the non-recursive model
  - $\rho_{k,j+m,j+m+n|j}$ by $\rho_{k,j+m,j+m+n|j}^{R}$ for the recursive model
- Then
  - $\rho_{k,j+m,j+m+n|j}^{NR} \geq \rho_{k,j+m,j+m+n|j}^{R}$
  - $\rho_{k,j+m,j+m+n|j}^{NR} / \rho_{k,j+m,j+m+n|j}^{R} \to 1$ as $j \to \infty$
Conclusion

• The recursive and non-recursive models considered here are quite different but generate identical (chain ladder) forecasts
• However their prediction errors differ
• How should one decide which of these chain ladder models to adopt?
• Correlation properties of forecasts might provide one criterion for the decision
  – e.g. if one wishes to assume heavy correlations, one might adopt the recursive form

Questions?