Chain ladder correlations

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Overview

- The chain ladder produces cell-by-cell forecasts of future claims experience
- There are two distinct families of model that support the chain ladder algorithm
- Consideration will be given to the correlations between the forecasts of different cells (conditional on information to date)
  - Separately for the two families
- Paper to appear shortly as:

  "Chain ladder correlations". Variance 5(2).

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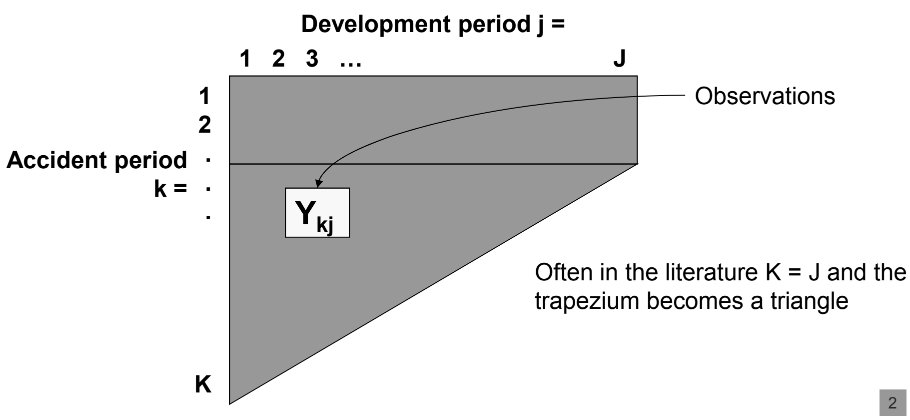
Framework and notation

- Claims reserving trapezium



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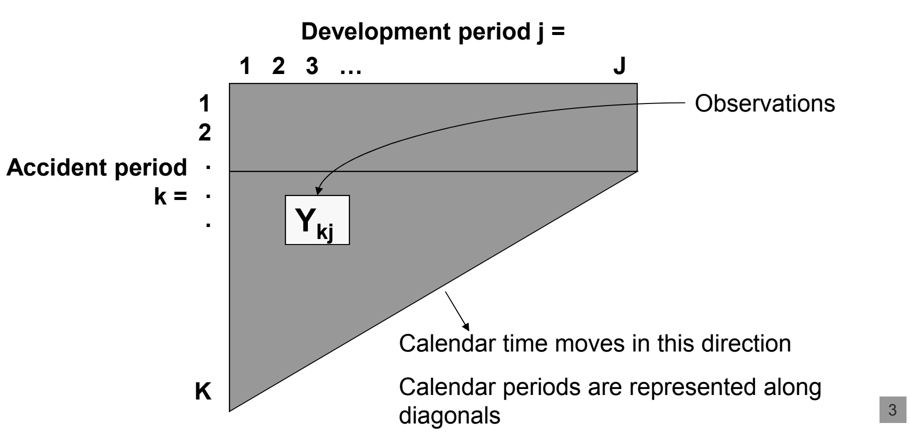
Framework and notation

- Claims reserving trapezium



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Framework and notation

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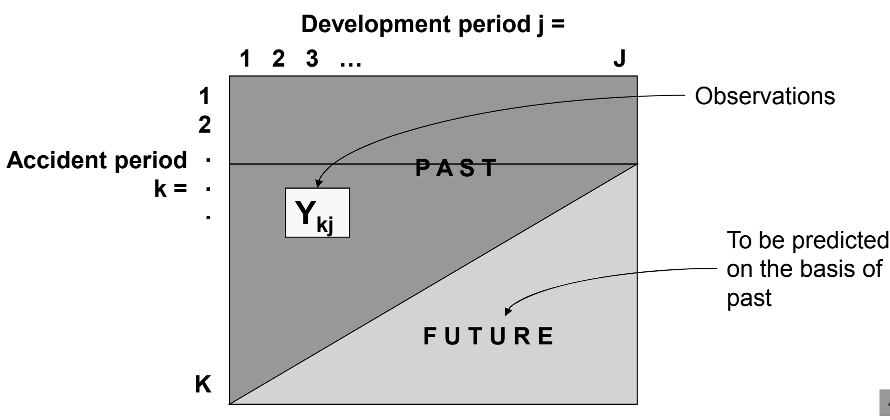

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Framework and notation

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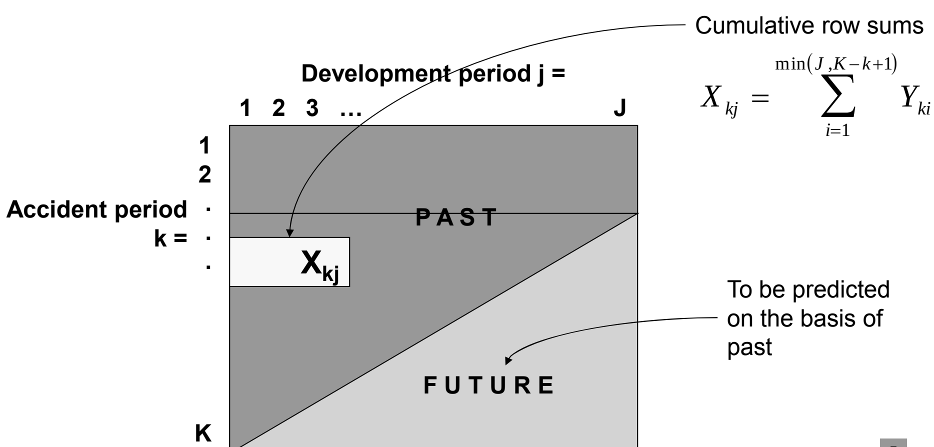

$$X_{kj} = \sum_{i=1}^{\min(J, K-k+1)} Y_{ki}$$

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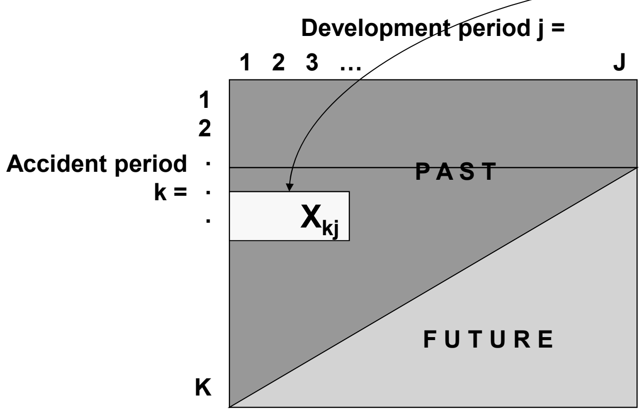
Chain ladder algorithm

- Claims reserving trapezium



Cumulative row sums

$$X_{kj} = \sum_{i=1}^{\min(J, K-k+1)} Y_{ki}$$

Calculate

$$\hat{f}_j = \sum_{k=1}^{K-j} X_{k,j+1} \Big/ \sum_{k=1}^{K-j} X_{kj}$$

Forecast

$$\hat{X}_{kj} = X_{k,K-k+1} \; \hat{f}_{K-k+1} \dots \hat{f}_{j-1}$$

↑ Last diagonal from the past

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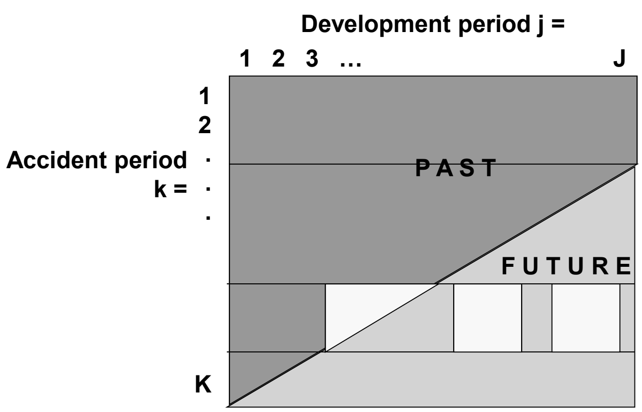
Correlations of predictions



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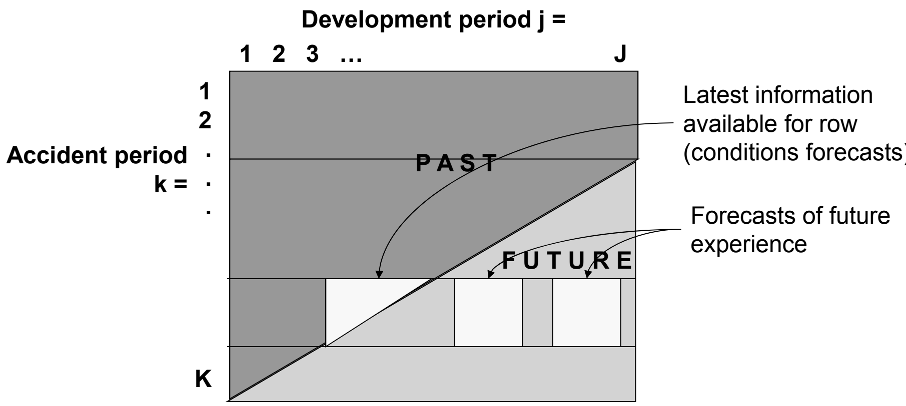
Correlations of predictions



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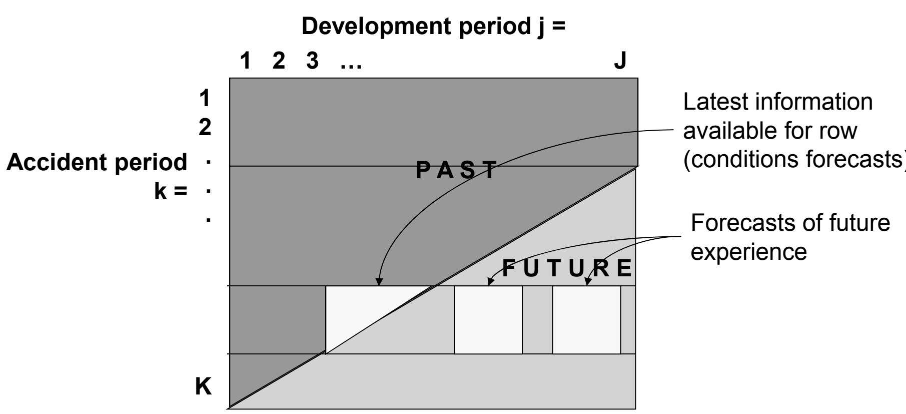
Correlations of predictions



Consider correlations of within-row forecasts conditioned on most recent information

Corr $[X_{k,j+m}, X_{k,j+m+n} | X_{kj}]$

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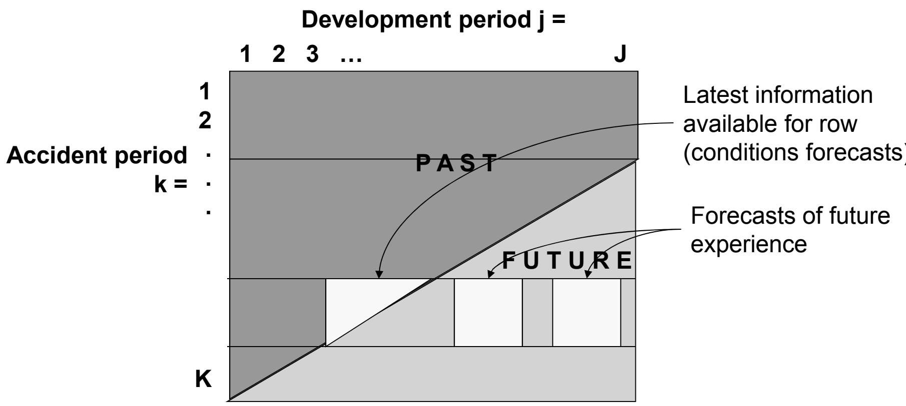
Correlations of predictions



Consider correlations of within-row forecasts conditioned on most recent information

Corr $[X_{k,j+m}, X_{k,j+m+n} | X_{kj}] = \rho_{k,j+m,j+m+n|j}$

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ODP Mack model

- **(ODPM1)** Accident periods are stochastically independent, i.e. Y_{kj}, Y_{hi} are independent if $k \neq h$
- **(ODPM2)** For each k the X_{kj} (j varying) form a Markov chain
- **(ODPM3)** For each k=1,2,…,K and j=1,2,…,J-1,

$$X_{k,j+1} | X_{kj} \sim ODP(f_j X_{kj}, \varphi_{j+1})$$

Parameters f_j are referred to as age-to-age factors

Example of a **recursive model**

Recursive because each observation depends on predecessor in same row

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ODP cross-classified model

- **(ODPCC1)** All random variables Y_{kj} are stochastically independent
- **(ODPCC2)** For each k = 1, 2, …, K and j = 1, 2, …, J,
  a) $Y_{kj} \sim ODP(\alpha_k\beta_j,\varphi_j)$
  b) $\sum^{J}_{j=1} \beta_j = 1$

Example of a **non-recursive model**

Non-recursive because each observation independent of predecessors in same row

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Relevance of ODP Mack and cross-classified models

- Very different models
- But in both cases chain ladder algorithm gives MLE forecasts
- But what about correlations of the forecasts under the respective models?

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Conditional correlations of predictors

- Explicit expressions for $\rho_{k,j+m,j+m+n|j} = \text{Corr}[X_{k,j+m}, X_{k,j+m+n} | X_{kj}]$ can be obtained (see paper)
- These are not especially informative in themselves
- Subsequent discussion concentrates on their properties

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Properties of conditional correlations

- **ODP Mack model**

- $\rho_{k,j+m,j+m+n|j} \downarrow$ as $n \uparrow$

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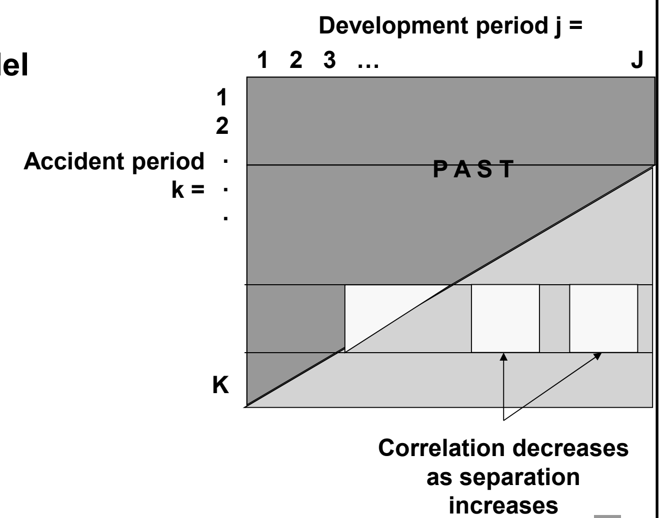
Properties of conditional correlations

- **ODP Mack model**

- $\rho_{k,j+m,j+m+n|j}$ ↓ as n ↑



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Properties of conditional correlations

- **ODP Mack model**

- $\rho_{k,j+m,j+m+n|j}$ ↓ as n ↑
- $\rho_{k,j+m,j+m+n|j}$ ↑ as any
  - φ_i ↑, i=j+1,…,j+m
  - φ_i ↓, i=j+m+1,…,j+m+n

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Properties of conditional correlations

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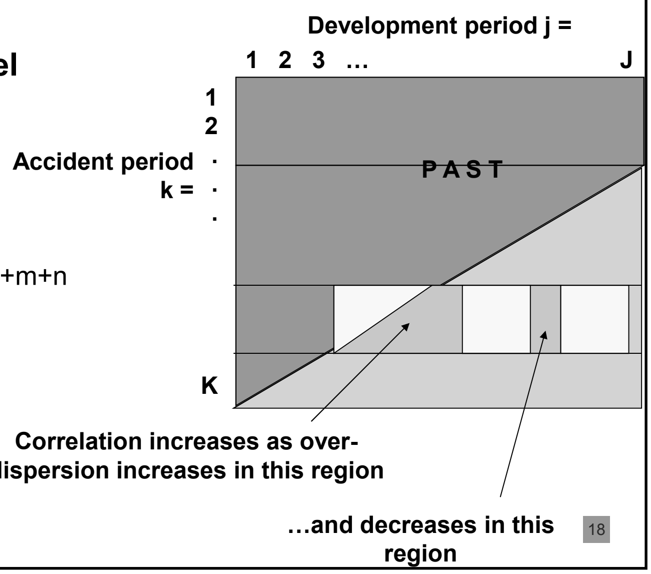

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Properties of conditional correlations

- **ODP Mack model**

- $\rho_{k,j+m,j+m+n|j}$ ↓ as n ↑
- $\rho_{k,j+m,j+m+n|j}$ ↑ as any
  - φ_i ↑, i=j+1,…,j+m
  - φ_i ↓, i=j+m+1,…,j+m+n
- $\rho_{k,j+m,j+m+n|j}$ ↑ as any f_i ↑, i=j+1,…,j+m+n **provided that**
  - $\varphi_i f_i /(f_i -1)$ ↑, i=j+1,…,j+m
  - $\varphi_i f_i /(f_i -1)$ ↓, i=j+m+1,…,j+m+n

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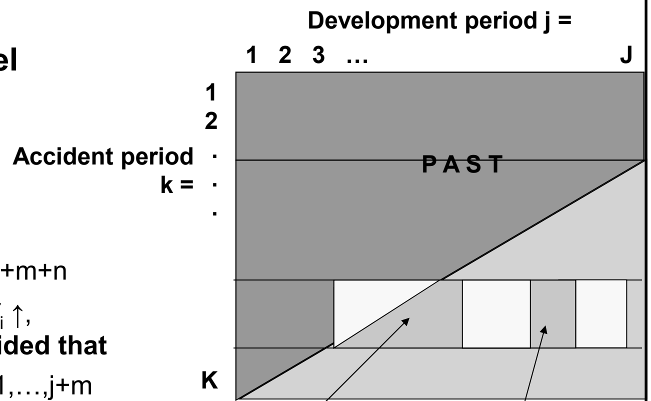
Properties of conditional correlations

- **ODP Mack model**

- $\rho_{k,j+m,j+m+n|j} \downarrow$ as $n \uparrow$
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any
  - $\varphi_i \uparrow$, $i=j+1,\dots,j+m$
  - $\varphi_i \downarrow$, $i=j+m+1,\dots,j+m+n$
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any $f_i \uparrow$, $i=j+1,\dots,j+m+n$ **provided that**
  - $\varphi_i f_i /(f_i -1) \uparrow$, $i=j+1,\dots,j+m$
  - $\varphi_i f_i /(f_i -1) \downarrow$, $i=j+m+1,\dots,j+m+n$



Correlation increases as claims development increases in this region

…and decreases in this region

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Properties of conditional correlations

- **ODP Mack model**

- $\rho_{k,j+m,j+m+n|j} \downarrow$ as $n \uparrow$
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any
  - $\varphi_i \uparrow$, $i=j+1,\dots,j+m$
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  - $\varphi_i f_i /(f_i -1) \downarrow$, $i=j+m+1,\dots,j+m+n$

- **ODP cross-classified model**

- $\rho_{k,j+m,j+m+n|j} \downarrow$ as $n \uparrow$
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any
  - $\varphi_i \uparrow$, $i=j+1,\dots,j+m$
  - $\varphi_i \downarrow$, $i=j+m+1,\dots,j+m+n$
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any
  - $\beta_i \uparrow$, $i=j+1,\dots,j+m$
  - $\beta_i \downarrow$, $i=j+m+1,\dots,j+m+n$

Again effect of increasing or decreasing claims development

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Claims development in recursive and non-recursive models

- Claims development effects of correlations expressed in terms of:
  - The f_i for recursive models
  - The β_i for non-recursive models
- **BUT** the two can be shown to be related:
  - $f_j = \sum_{i=j}^{j+1} \beta_i \,/\, \sum_{i=j}^{j} \beta_i$
- Then…

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Properties of conditional correlations (cont'd)

- **ODP Mack model**
- $\rho_{k,j+m,j+m+n|j} \downarrow$ as n $\uparrow$
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any
  - $\varphi_i \uparrow$, i=j+1,…,j+m
  - $\varphi_i \downarrow$, i=j+m+1,…,j+m+n
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any $f_i \uparrow$, i=j+1,…,j+m+n **provided that**
  - $\varphi_i f_i /(f_i -1) \uparrow$, i=j+1,…,j+m
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- **ODP cross-classified model**
- $\rho_{k,j+m,j+m+n|j} \downarrow$ as n $\uparrow$
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any
  - $\varphi_i \uparrow$, i=j+1,…,j+m
  - $\varphi_i \downarrow$, i=j+m+1,…,j+m+n
- $\rho_{k,j+m,j+m+n|j} \uparrow$ as any $f_i \uparrow$, i=j+1,…,j+m+n **provided that**
  - $\Phi_{i+1} f_i /(f_i -1) \uparrow$, i=j+1,…,j+m
  - $\Phi_{i+1} f_i /(f_i -1) \downarrow$, i=j+m+1,…,j+m+n

Now almost identical

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Comparison between correlations of recursive and non-recursive models (1)

- Consider the case of recursive and non-recursive models with common values of the f_i, φ_i
- The models have been seen to have similar ordering properties
- Are they actually different?

Comparison between correlations of recursive and non-recursive models (2)

- Denote
  - $\rho_{k,j+m,j+m+n|j}$ by $\rho^{NR}_{k,j+m,j+m+n|j}$ for the non-recursive model
  - $\rho_{k,j+m,j+m+n|j}$ by $\rho^{R}_{k,j+m,j+m+n|j}$ for the recursive model
- Then
  - $\rho^{NR}_{k,j+m,j+m+n|j} \geq \rho^{R}_{k,j+m,j+m+n|j}$
  - $\rho^{NR}_{k,j+m,j+m+n|j} / \rho^{R}_{k,j+m,j+m+n|j} \rightarrow 1$ as $j \rightarrow \infty$

Conclusion

- The recursive and non-recursive models considered here are quite different but generate identical (chain ladder) forecasts
- However their prediction errors differ
- How should one decide which of these chain ladder models to adopt?
- Correlation properties of forecasts might provide one criterion for the decision
  - e.g. if one wishes to assume heavy correlations, one might adopt the recursive form

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Questions?

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