Multi-State Microeconomic Model for Pricing and Reserving a disability insurance policy over an arbitrary period

Benjamin Schannes – April 4, 2014
Some key disability statistics:

- One disabling accident per second: US
- One disabling illness per 2 seconds: UK, Canada, France
Motivation and Setting

• The universal trigger event for Disability Insurance = the inability to work
• Compensation systems:
  - Public health insurance
  - Private health care coverage:
    ✓ Group insurance
    ✓ Individually purchased
• Group insurance « paradox »
  - Many different risk profiles
  - But Often Uniform Premium = Risks averaged

Key idea to simplify risk assessment

Make as if many identical individuals: Representative Insured (RI) Assumption

Covered Risk = Aggregate Risk

Multistate Modeling
Overview of the Multi-State Model

<table>
<thead>
<tr>
<th>Transition</th>
<th>Depends on</th>
<th>Transition</th>
<th>Modeling</th>
<th>Resulting Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disablement</td>
<td>Age</td>
<td></td>
<td></td>
<td>locally time-homogeneous markov renewal</td>
</tr>
<tr>
<td>Recovery</td>
<td>Age and Duration</td>
<td></td>
<td></td>
<td>locally time-homogeneous semi-markov</td>
</tr>
<tr>
<td>Death of an active life</td>
<td>Age</td>
<td></td>
<td></td>
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<tr>
<td>Death of a disabled life</td>
<td>Age</td>
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</tr>
</tbody>
</table>
Estimation and Graduation of Transition Intensities

- **Disablement**
  - Poisson coefficients $\hat{\theta}$
  - $\lambda_{ai}(t|Z = z) = \exp(\hat{\theta}'z)$

- **Recovery**
  - Cox Coefficients $\hat{\beta}$
  - Baseline Hazard $\lambda_{0,ia}(\cdot)$
  - Frailty $\nu$
  - $\lambda_{ia}(t|Z = z) = \lambda_{0,ia}(t) \exp(z'\hat{\beta})\nu$

- **Mortality**
  - Annual Mortality rates $q_x$
  - $\hat{\lambda}_{i,a,d}(x \times 365.25) = -\frac{\ln(1-q_x)}{365.25}$
Application

• Representative insured
  - Male
  - 25
  - Large City
  - Finance & Insurance
  - $ 65,000

• Main conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deferred Period</td>
<td>91 Days</td>
</tr>
<tr>
<td>Targeted replacement rate a</td>
<td>85 %</td>
</tr>
<tr>
<td>Maximum Benefit Amount a x $450,000</td>
<td></td>
</tr>
<tr>
<td>State-guaranteed minimum replacement rate</td>
<td>50%</td>
</tr>
</tbody>
</table>

• In the simplest case

\[ B(s, t) = a \times \text{Salary} \times (t - s) \]
Simulation Results: Summary

### Empirical Distribution of the Discounted Cost

![Graph showing empirical distribution of the discounted cost](image)

### Summary

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Variable</th>
<th>Aggregate Duration of Disability Spells</th>
<th>Total Discounted Cost of DI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
<td>247.2</td>
<td>6,582.62</td>
</tr>
<tr>
<td>Std Dev.</td>
<td></td>
<td>205.4</td>
<td>14,446.32</td>
</tr>
<tr>
<td>Skewness</td>
<td></td>
<td>2.22</td>
<td>3.79</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td>2,684.3</td>
<td>258,342.07</td>
</tr>
</tbody>
</table>

- Deferred Period
- Coefficient of Variation > 2
Towards a simple technical account

- Modified Standard Deviation Principle (MSDP) consistent with the assumption (RI)
  - Aggregate Cost $S_n$
  \[ \Pi(S_n) = E[S_n] + \xi \sigma(S_n) \] (MSDP)
- The following convergence holds
  \[ S_n - \Pi(S_n) \xrightarrow{d} \mathcal{N}(-\xi, 1) \]

<table>
<thead>
<tr>
<th>Time</th>
<th>y = 0</th>
<th>y =1−</th>
<th>y =2−</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>271.25</td>
<td>279.39</td>
<td>567.16</td>
</tr>
<tr>
<td>Reserves</td>
<td>(117.63)</td>
<td>55.53</td>
<td>235.41</td>
</tr>
<tr>
<td>Claims paid</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Profit</td>
<td>388.88</td>
<td>223.86</td>
<td>331.75</td>
</tr>
</tbody>
</table>

Scenario:
- No waiver of premiums
- No disability > deferred period the first 2 years
- Risk horizon: retirement
- 99.5% solvency constraint
Conclusions and extensions

• Extensions
  ➢ Deviations from the rescaled limit distribution
  ➢ Optimal Representative Insured
  ➢ Heterogeneous insured models

(RI) assumption, although apparently rough, simplifies the Multi-State Model and facilitates risk management. We get more accurate and consistent pricing and reserving.
References


Thank you for your attention!
Appendix
Multi-State Model: a trajectory example

States

- 1 Active
- 2 Disabled
- 3 Dead

Disabled to Dead: Depends on age
Active to Dead: Depends on age
Recovery: Depends on age and Duration
Disabled to Dead: Depends on age
Active to Dead: Depends on age

Time of Disablement
Time of Recovery or Death
Time of Death
Probabilistic Framework

• Let \( \{(X_t, D_t), t \geq 0\} \) be a bivariate process where \( X_t \) is the state occupied at time \( t \) (right-continuous paths) and \( D_t \) is the duration of stay in this state.

• **Markov disablement process:** the instantaneous transition rate from the ‘active’ state to the ‘disabled’ state depends only on the age

\[
\lambda_{ai}(x + t) = \lim_{\Delta t \downarrow 0} \frac{\Pr[X_{t+\Delta t} = i \mid X_t = a]}{\Delta t}
\]

• **Semi-Markov Recovery process:** depends both on the age and the duration of the current instance of disability

\[
\lambda_{ia}(x + t; s) = \lim_{\Delta t \downarrow 0} \frac{\Pr[X_{t+\Delta t} = a \mid X_t = i, D_t = s]}{\Delta t}
\]

• **Mortality intensities from the disability state and from the ‘active’ state:** equal and Markov

\[
\lambda_{id}(x + t) = \lambda_{ad}(x + t)
\]

Different assumptions drive transitions and require a modeling specific to each type of transition.
Reserves Dynamics

- **Thiele’s differential equation for the Active Prospective Reserve**
  
  For an insured aged $x$ at policy issue, we have at time $t \notin \text{Disc}(\Pi_a) = \{t_a,0, t_{a,1}, ..., t_{a,q}\}$
  
  $$dV_a(t) = r(t) V_a(t) \, dt + \pi_a(t) \, dt - \lambda_a(x + t) \, dt \left( c_{ai}(t) + V_i(t, 0) - V_a(t) \right) + \lambda_{ad}(x + t) \, dt \, V_a(t)$$

  where $t \mapsto \Pi_a(t)$ is a right-continuous and non-decreasing premium process, and $t \mapsto c_{ai}(t)$ is a lump sum in case of transition to disability.
  
  The solution is uniquely determined with the conditions
  
  $$V_a(t_{a,j}) = V_a(t_{a,j}^-) + \Delta \Pi_a(t_{a,j}), \quad j = 0,1, ..., q$$

- **Thiele’s differential equation for the Disabled Prospective Reserve**
  
  For an insured aged $x$ at policy issue, disabled at time $t \notin \text{Disc}_s(B_i) = \{t_i,s,0, t_{i,s,1}, ..., t_{i,s,q} \}$ with duration $s$ since the disability onset
  
  $$d_t V_i(t, s) = r(t) V_i(t, s) \, dt - d_s V_i(t, s) - b_i(t, t - s) \, ds - \lambda_{ia}(x + t, s) \, dt \left( V_a(t) - V_i(t, s) \right) + \lambda_{id}(x + t, s) \, dt \, V_i(t, s)$$

  where $(t, t - s) \mapsto B_i(t, t - s)$ is a right-continuous and non-decreasing benefit process, well-defined for $t \geq s$.
  
  Again, the solution is uniquely determined with the conditions
  
  $$V_i(t_{i,s,j}) = V_i(t_{i,s,j}^-) - \Delta B_i(t_{i,s,j}, t_{i,s,j} - s), \quad j = 0,1, ..., q_s$$

Thiele’s equations exhibit positive and negative contributions to the reserve, which make intuitive sense.