Optimal Policyholder Behavior for Withdrawal Guarantees in Variable Annuities

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Introduction

Policyholder Behavior
VA + GMWB: A Simple Example
How do you price a financial option?
Are VA policyholders value-maximizers?

How do Market Frictions Affect Optimal W/d Behavior?

Does Market Incompleteness Play a Role As Well?

Subjective Mortality Risk

Can VA Guarantees Have Negative Prices?

Conclusion and Outlook
Many modern life insurance products depend on policyholder behavior

- Surrender options
- Withdrawal guarantees (GMWBs) in Variable Annuities (VAs)
- Implicit option to (re-)allocate money in different subaccounts

But: Policyholder behavior is not well understood

- Relatively new products $\implies$ Lack of data
- Many insurers suspended their VA business in recent years
  - Or made substantial adjustments to its guarantees

Moody’s (June 2013):

"Unpredictable policyholder behavior challenges US life insurers’ variable annuity business"

Today: Policyholder behavior for withdrawal guarantees in VAs
VA + GMWB: A Simple Example

- Policyholder invests $100,000 in VA for 15 years
  - Money invested in mutual fund
  - Adds on a **Guaranteed Minimum Withdrawal Benefit (GMWB)**
    - PH has the right (but no obligation) to withdraw $7K each year
    - If VA account depletes, withdrawal amount comes out of insurer’s pocket
    - Until $100K have been withdrawn on aggregate
    - PH can withdraw more than $7K, if account value permits
    - But guarantee covers only $7K per year
  - Pays $X basis points (as % of account value) annually for this guarantee
  - At death: beneficiaries receive account value
  - If alive at maturity: PH receives account value

- Can we find the fair guarantee fee $X$?
  - $X$ depends on withdrawal behavior !!
    - If PH withdraws less $\implies$ Insurer less likely to make payment
    - If PH withdraws less $\implies$ Insurer collects more fees
Introduction
Policyholder Behavior for GMWBs – What can we do about it?

- Find withdrawal strategy that is **financially optimal**
  - Similar to pricing/early exercise of American options
    - Continuation value vs. exercise value
    - Choose withdrawal amount that maximizes w/d amount + VA continuation value
    - Recursively, year by year
  - Problem: fair GMWB fee way above what is charged in practice

- Behavioral Economics
  - Young science
  - Lots of (different) opinions
  - Theory not well developed yet
  - Not much help (yet!) for a product this complicated

- **Our approach**: Find a middle ground . . .
  - How should policyholder withdraw optimally, under various conditions?
How do you price a financial option?

- **Textbook:**
  - Use **Arbitrage Pricing**
    - Option price = initial value of replicating portfolio (→ e.g. Black-Scholes)
  - (If given a choice:) When should you exercise your option?
    - When exercise value > continuation value !!
  - Requires a complete, frictionless market
    - All assets can be traded at competitive market prices
    - No transaction costs, no taxes

- Are VA policyholders value-maximizers?
Introduction

Are VA policyholders value-maximizers?

Why wouldn’t they be?

- VAs cannot be sold in the market, cannot be “split up”
- VAs grow tax-deferred
  - Replicating portfolio does not
  - This is why people buy VAs, so it might impact their withdrawal decisions

How can we model this?

- Market frictions only
  - Subjective Risk-Neutral Valuation
    - PH withdraws in order to maximize expected after-tax payout
- Market frictions and market incompleteness
  - Life-Cycle Model
    - PH withdraws in order to maximize expected lifetime utility of consumption
How do Market Frictions Affect Optimal W/d Behavior?

1. Introduction

2. How do Market Frictions Affect Optimal W/d Behavior?
   - Risk-Neutral Valuation from Policyholder's Perspective
   - Parameter Assumptions
   - Optimal Withdrawal Behavior
   - Sensitivities
   - Analysis of an Empirical Product

3. Does Market Incompleteness Play a Role As Well?

4. Subjective Mortality Risk

5. Can VA Guarantees Have Negative Prices?

6. Conclusion and Outlook
How do Market Frictions Affect Optimal W/d Behavior?
Risk-Neutral Valuation from Policyholder’s Perspective

- Under standard RNV, withdrawing always optimal; **but**:
  - ▶ VAs popular because of preferential tax treatment
  - ⇒ Taxes might impact withdrawal decisions

- Tradeoff with taxes: Withdrawing means . . .
  - ⊕ Making use of guarantee
  - ⊕ Reducing fee payments
  - ⊖ Foregoing tax benefits

⇒ Develop **"subjective" risk-neutral valuation (SRNV)** approach
  - ▶ Takes into account differences in taxation

- When cash-flow is taxed differently than replicating portfolio:
  - ▶ Ross (JPE, 1986): No universal pricing measure exists
  - ▶ Valuation of cash-flows *locally* (i.e. agent-specific / subjective)
How do Market Frictions Affect Optimal W/d Behavior?

Subjective Risk-Neutral Valuation (SRNV) approach

Determine time-$t$ value ($X_t$) of post-tax cash flow $X_{t+1}$

- Define $X_t$ as amount needed in replicating portfolio (at time $t$)
  - ... to attain $X_{t+1}$ at time $t + 1$ after taxes
  - Gains in replicating PF taxed at rate $\kappa$
  - Assume complete pre-tax market

- For given (assumed) value of $X_t$:
  - Find pre-tax cashflow $Y_{t+1}$ that yields $X_{t+1}$ after taxes
  - “Discount” $Y_{t+1}$ to time $t$ with (unique) pre-tax measure $\mathbb{Q}$

- Iterate over $X_t$

Proposition 1.

Any post-tax cash flow $X_{t+1}$ can be valued uniquely at time $t$ as $X_t$, where

$$X_t \cdot e^r = E_t^\mathbb{Q} [X_{t+1}] + \frac{\kappa}{1 - \kappa} \cdot E_t^\mathbb{Q} [\max\{X_{t+1} - X_t, 0\}]$$
Implement using **recursive dynamic programming**

- For all times and states, (recursively) determine optimal w/d amount $w_t$
- To maximize expected after-tax payout from the VA:

$$V_t(y_t) = \max_{w_t} \left[ w_t - (\text{fees+taxes}) + V_{t+1} \right], \quad \text{(1)}$$

* $y_t$: time-$t$ state vector

- where the continuation value $V_{t+1}$ is given implicitly by

$$V_{t+1} \cdot e^r = E_t^Q[Y] + \frac{\kappa}{1 - \kappa} \cdot E_t^Q[\max\{ Y - V_{t+1}, 0 \}], \quad \text{(2)}$$

* $r$: risk-free interest rate  
* $\kappa$: capital gains tax rate

- and where

$$Y = q_{x+t} \cdot b_{t+1} + p_{x+t} \cdot V_{t+1}(y_{t+1}). \quad \text{(3)}$$

* $b_{t+1}$: time $t + 1$ death benefit payment
To implement the “simple example” from above (benchmark case):

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Policyholder &amp; contract specification</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age at inception</td>
<td>$x$</td>
<td>55</td>
</tr>
<tr>
<td>VA principal</td>
<td>$P_0$</td>
<td>100,000</td>
</tr>
<tr>
<td>Years to maturity</td>
<td>$T$</td>
<td>15</td>
</tr>
<tr>
<td>Annual guaranteed amount</td>
<td>$g^W$</td>
<td>7,000</td>
</tr>
<tr>
<td>Excess withdrawal fee</td>
<td>$s_t$</td>
<td>8%, 7%, . . . , 1%, 0%, 0%, . . .</td>
</tr>
<tr>
<td><strong>Financial market parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>0.05</td>
</tr>
<tr>
<td>Volatility</td>
<td>$\sigma$</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>Tax rates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income tax rate</td>
<td>$\tau$</td>
<td>30%</td>
</tr>
<tr>
<td>Capital gains tax rate</td>
<td>$\kappa$</td>
<td>23%</td>
</tr>
<tr>
<td>Early withdrawal penalty</td>
<td>$s^g$</td>
<td>10%</td>
</tr>
</tbody>
</table>
How do Market Frictions Affect Optimal W/d Behavior?

**Optimal Withdrawal Behavior**

Benchmark Case: $t = 10$, $G_{10} = 100$, $H_{10} = 100$ (in $1,000$)

<table>
<thead>
<tr>
<th></th>
<th>SRNV</th>
<th>RNV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^Q[\text{Fees}]$</td>
<td>2,555</td>
<td>1,394</td>
</tr>
<tr>
<td>$E^Q[\text{GMWB}]$</td>
<td>2,498</td>
<td>4,059</td>
</tr>
<tr>
<td><strong>Insurer’s Profit</strong></td>
<td>57</td>
<td>-2,665</td>
</tr>
<tr>
<td><strong>Agg. Withdrawals</strong></td>
<td>5,260</td>
<td>265,870</td>
</tr>
</tbody>
</table>

- Either way: withdraw when account goes down
- With taxes: no surrender, even when guarantee is worthless

⇒ With tax considerations: insurer collects more fees
  - Can charge less: $X = 20$ bps (without taxes: 64 bps)

∽ Taxation clearly matters!
### Sensitivities

- **Fair GMWB fee (in bps) for different market parameters**
  - Fair fee without taxes in parentheses
  
<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>16%</th>
<th>19%</th>
<th>22%</th>
</tr>
</thead>
<tbody>
<tr>
<td>3%</td>
<td>35 (105)</td>
<td>55 (146)</td>
<td>79 (198)</td>
</tr>
<tr>
<td>5%</td>
<td>11 (43)</td>
<td>20 (64)</td>
<td>31 (89)</td>
</tr>
<tr>
<td>7%</td>
<td>3 (18)</td>
<td>7 (30)</td>
<td>13 (45)</td>
</tr>
</tbody>
</table>

- **Fair GMWB fee (in bps) for different tax rates**
  - Fair fee without taxes: 64 bps
  
<table>
<thead>
<tr>
<th>$\tau$</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>20</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>23%</td>
<td>17</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>25%</td>
<td>15</td>
<td>17</td>
<td>20</td>
</tr>
</tbody>
</table>
How do Market Frictions Affect Optimal W/d Behavior?

Analysis of an Empirical Product

- Implement VA offered in U.S. market
  - ASL II by Prudential Annuities Life Assurance Corporation

- Key differences to simple GMWB example
  - Charges of 165 bps (of account value) p.a. (for M&E risk and Admin.)
  - Basic death benefit included
  - GMWB eligible for additional 35 bps p.a.
    - Includes step-up option
    - At maturity or death of PH: option to receive remaining benefits base, annuitized with zero interest
    - Guarantee fee waived after 7 years, if no withdrawals are made
  - Investment in riskiest eligible fund: Pro Fund VP Bull
    - Returns similar to S&P500

- Implement optimization with SRNV approach
Valuation Results for *ASL II*:

<table>
<thead>
<tr>
<th></th>
<th>With GMWB</th>
<th>W/o GMWB</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMWB fees collected</td>
<td>3,473</td>
<td>3,473</td>
<td></td>
</tr>
<tr>
<td>Other fees collected</td>
<td>25,053</td>
<td>22,242</td>
<td>2,811</td>
</tr>
<tr>
<td>Costs of guarantees</td>
<td>7,541</td>
<td>2,866</td>
<td>4,675</td>
</tr>
<tr>
<td>Insurer's profit (NPV)</td>
<td>20,985</td>
<td>19,376</td>
<td>1,609</td>
</tr>
<tr>
<td>Years under contract</td>
<td>20.37</td>
<td>16.01</td>
<td></td>
</tr>
<tr>
<td>Surrender rate</td>
<td>&lt; 0.01%</td>
<td>41.1%</td>
<td></td>
</tr>
<tr>
<td>$V_0$</td>
<td>101,574</td>
<td>100,859</td>
<td></td>
</tr>
<tr>
<td>$V_0$ without taxes</td>
<td>99,053</td>
<td>98,420</td>
<td></td>
</tr>
</tbody>
</table>

- Marginal value of GMWB to insurer: $1,609
- Guarantee *not* under-priced
- Without tax considerations, VA not worth buying
Does Market Incompleteness Play a Role As Well?

1 Introduction

2 How do Market Frictions Affect Optimal W/d Behavior?

3 Does Market Incompleteness Play a Role As Well?
   A Life-Cycle Model
   Optimal Withdrawal Behavior – Preliminary Results

4 Subjective Mortality Risk

5 Can VA Guarantees Have Negative Prices?

6 Conclusion and Outlook
Does Market Incompleteness Play a Role As Well?

A Life-Cycle Model

- Frictions matter, but what about incompleteness?
  - Need to build a bigger (economic) model...

- Develop life-cycle model:
  - Risk-averse policyholder maximizes life-time utility
  - Can invest in outside account
  - Drawbacks:
    - Complex model, requires simplifying assumptions
    - Not preference independent

- Bellman Equation:

\[
V_t(y_t) = \max_{C_t, w_t, \nu_t} u_C(C_t) + \beta \cdot \mathbb{E}^P_t \left[ q_{x+t} \cdot u_B \left( b_{t+1} \left| \frac{S_{t+1}}{S_t} \right. \right) + p_{x+t} \cdot V_{t+1} \left( y_{t+1} \left| \frac{S_{t+1}}{S_t} \right. \right) \right]
\]

- ...subject to a whole bunch of constraints...
- Solve (again) by recursive dynamic programming
Does Market Incompleteness Play a Role As Well?

Optimal Withdrawal Behavior – Preliminary Results

• PH behaves very similar to SRNV model
  ▶ Preferences have little impact
    ★ PH can attain desired risk exposure by adjusting outside account
    ★ Outside investment opportunity “completes market”
  ⇒ Optimal behavior driven by (subjective) value maximization

• One source of market incompleteness remains: Biometric risk
  ▶ Not very significant for GMWBs (age of PH: 55–70)
  ▶ Even less relevant if PH has access to life-contingent products
  ▶ Markets more incomplete for older ages / unlimited durations
    ★ E.g., lifetime withdrawal guarantees
    ★ Pension annuities offer protection against biometric risk
    ★ But don’t protect simultaneously against long-tailed biometric & investment risk
Subjective Mortality Risk

Introduction

How do Market Frictions Affect Optimal W/d Behavior?

Does Market Incompleteness Play a Role As Well?

Subjective Mortality Risk

- Literature Summary
- Implementation
- Impact on Optimal Withdrawal Behavior

Can VA Guarantees Have Negative Prices?

Conclusion and Outlook
Subjective Mortality Risk

- GMWB valuable only while policyholder is alive
- Withdrawal behavior depends on PH’s perception of his/her mortality risk
  - Even in SRNV model:
    \[ V_t(y_t) = \max_{w_t} [w_t - (\text{fees+taxes}) + V_{t+}^+] , \]
    where the continuation value \( V_{t+}^+ \) is given implicitly by
    \[ V_{t+}^+ \cdot e^r = \mathbb{E}_t^Q[Y] + \frac{\kappa}{1 - \kappa} \cdot \mathbb{E}_t^Q[\max\{Y - V_{t+}^+, 0\}] , \]
    and where
    \[ Y = q_{x+t}^{\text{subj}} \cdot b_{t+} + p_{x+t}^{\text{subj}} \cdot V_{t+1}(y_{t+1}) . \]
- Since payouts in “death” and “alive” state differ, optimal \( w_t \) depends on probability weight that PH assigns to each state
Subjective Mortality Risk

- Undergraduate research project (Summer 2013)
  - Impact of subjective mortality risk on policyholder behavior
  - *University of St. Thomas* undergrads: Clem Foltz, Nathan Kent, Yabing Yang
  - Sponsored by National Science Foundation (CSUMS grant)

- How do people’s subjective mortality perceptions differ from objective mortality risk?
  - Data sources:
    - *Health and Retirement Study* (HRS)
    - *Survey of Health, Aging, and Retirement in Europe* (SHARE)
    - Individual surveys (e.g. Harrison & Rutström, 2006; Jarnebrant & Myrseth, 2013)
  - Academic studies in demography & economics literature
Subjective Mortality Risk

Literature Summary

- Difficulties understanding and quantifying one’s own mortality risk
  - Poor understanding of the concept of Probability
  - One’s mortality is harder to visualize than other probabilistic events
  - Lack of experience (Harrison and Rutström, 2006)
  - Large amount of focal responses (“0”, “.5”, and “1”)

- Substantial Heterogeneity in Subjective Mortality Beliefs
  - Gender gap
    - Males slightly overestimate their survival probabilities to age 75, while females significantly underestimate theirs.
  - Other factors:
    - Cognitive abilities, socio-economic status, health, education, ethnicity, marital status, etc. beyond objective differences (Hurd and McGarry, 1995, 2002; Peracchi and Perotti, 2012)
Subjective Mortality Risk

Literature Summary

• Variation in systematic ways
  ▶ Flatness bias / constant hazard rate
    ✫ Individuals tend to underestimate their survival probability to age 75, but overestimate their survival to age 85 (Hamermesh, 1985; Elder, 2013)
  ▶ Optimism & pessimism
    ✫ “Much of the heterogeneity in subjective survival risks is related to a general optimism/pessimism factor.” (Hill et. al, 2004)
  ▶ Longevity risk
    ✫ It’s difficult to predict medical advances and quantify mortality improvements
  ▶ Population averages
    ✫ People tend to absorb information from the entire population without accounting for individual characteristics (Hurd and McGarry, 2002; Andersson, 2011)
  ▶ Equal survival rates across time
    ✫ People in particular age groups across time might have a similar framework for thinking about mortality (Elder, 2007)
Subjective Mortality Risk

Implementation

- Objective mortality based on: Annuity 2000 Basic Table (ABT)
  - Based on HRS 2002 data
  - Life-table survival rate: 59% (for a person aged 50 to 64, with target age 75)
  - Average subjective survival response: 66% . \( q_{sub}^{subj} = 0.83 \times q_{ABT}^{ABT} \)
  - Mode of the “Optimist’s Beliefs”: 80%. \( q_{opt}^{opt} = 0.48 \times q_{sub}^{subj} \)
  - Mode of the “Pessimist’s Beliefs”: 46%. \( q_{pess}^{pess} = 1.29 \times q_{sub}^{subj} \)
- Elder (2013)
  - Based on HRS 2006 data
  - Average subjective survival response: 59.1% (life table: 67.56%)
  - Constant subjective hazard: \( q_{sub}^{subj} = 1.691\% \)
- More extreme beliefs:
  - Focal response of 100% survival rate: \( q_{sub}^{subj} = 0 \)
  - Highly pessimistic (hypochondriac) person \( q_{hypoch}^{hypoch} = 7.12 \times q_{sub}^{subj} \)
    - Not likely to purchase GMWB.
Subjective Mortality Risk
Impact on Optimal Withdrawal Behavior

Table: Valuation results based on Hill, Perry and Willis (2004) (φ = 19 bps).

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fees collected</td>
<td>2,444.6</td>
<td>2,508.5</td>
<td>2,500.3</td>
<td>2,504.7</td>
<td>2,507.8</td>
<td>2,534.4</td>
</tr>
<tr>
<td>Costs of GMWB</td>
<td>2,441.8</td>
<td>2,589.9</td>
<td>2,648.9</td>
<td>2,611.3</td>
<td>2,591.8</td>
<td>2,276.8</td>
</tr>
<tr>
<td>Insurer's profit</td>
<td>2.8</td>
<td>-81.4</td>
<td>-148.6</td>
<td>-106.6</td>
<td>-84.0</td>
<td>257.6</td>
</tr>
</tbody>
</table>

- Subjective mortality beliefs have minor impact
- But: Tend to *reduce* insurer’s profit
  - More optimistic policyholder has more incentives to withdraw
  - Investors pessimistic about their mortality unlikely to purchase GMWB
  - Over- and under-estimations do not cancel out
- Increasing guarantee fee by 1-2 bps seems sufficient
  - Perhaps more in utility-based framework (→ bequest motive)
  - Add a death benefit guarantee
Introduction

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Does Market Incompleteness Play a Role As Well?

Subjective Mortality Risk

Can VA Guarantees Have Negative Prices?

Conclusion and Outlook
Can VA Guarantees Have Negative Prices?

- **Arbitrage Pricing Theory**: An option cannot have a negative price!
  - Holder can always choose to ignore option
  - Issuer has nothing to gain, should charge positive price

- Result breaks down for products with preferential tax treatment
  - Valuation of PH and insurer no longer opposites
  - Third party involved: tax collector

- Example: VA + GMWB
  - Consider adding death benefit guarantee (GMDB)
    - At no extra charge !!
    - Reduces incentives to withdraw / surrender policy
    - Good for insurer: more fees, less guarantee!
    - Also: Delaying / foregoing withdrawals reduces tax payments
  - Both policyholder and insurer may be strictly better off
    - At “expense” of government
Can VA Guarantees Have Negative Prices?

Implications

- Not just a blackboard curiosity
  - 2-period model
  - Prudential’s ASL II

- Insurer willing to give away GMDB for free
  - In competitive insurance market: price of GMDB could be negative!
  - Might explain why GMDBs are now standard features of most VAs

- “New” role for life insurers
  - Design long-term savings products that best take advantage of investors’ tax benefits
    - Insurer and PH can “share” the tax savings
  - Financially savvy policyholders more profitable to insurers??
  - Lots of $$ to be made 😊
Conclusion and Outlook

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To sum up: Looking at withdrawal guarantees in VAs, we understand (a little better) what factors drive optimal policyholder behavior:

- **Subjective value maximization**
  - We develop valuation framework
  - Tractable & preference independent

- Can cause some guarantees to have negative prices in equilibrium
  - Might (partially) explain why GMDBs are now standard in most VAs

- Unobservable PH characteristics don’t matter too much
  - Risk aversion; marginal tax rates; etc.

Future research: When is market incompleteness important?

- For lifetime withdrawal guarantees??
- Can we find a “measure” for the incompleteness of savings products?
THANK YOU!

Questions?
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