Stress scenario generation for solvency and risk management

Marcus C. Christiansen. University of Ulm.
Lars Frederik Brandt Henriksen. University of Copenhagen.
Kristian Juul Schomacker. Edlund A/S.
Mogens Steffensen. University of Copenhagen.
Outline

• Worst case scenario
• Relation to VaR and Solvency II
• Finding the worst case scenario
• Examples
• Conclusion
Worst case scenario

- Worst case scenario
  \[ \tilde{\mu} = \arg\max_{m \in M} \{ V_{j_0}(t_0; m) \} \]
- \( V_{j_0}(t_0; m) \) is the reserve in state \( j_0 \) at time \( t_0 \)
- The \( m \in M \) are the possible future biometrical scenarios (of transition intensities \( t \rightarrow m_{jk}(t) \))
- \( V_{j_0}(t_0; \tilde{\mu}) = \sup_{m \in M} \{ V_{j_0}(t_0; m) \} \)
  if \( \tilde{\mu} \) is a worst case scenario
Applications of worst case scenarios

- Solvency II standard formula for life (assuming assets unaffected)
  \[ SCR_{life} = \text{VaR}_{0.995}(V_{j_0}(t_0; \mu) - V_{j_0}(t_0; \mu^{BE})) \]
  \( \mu \): Stochastic transition intensities
  \( \mu^{BE} \): Deterministic best estimates transition intensities

- \[ SCR_{life} \leq \sup_{m \in M}\{V_{j_0}(t_0; m)\} - V_{j_0}(t_0; \mu^{BE}) \]
  if \( P(\mu \in M) \geq 0.995 \)

- Worst case scenarios can also be used for premium settlement of traditional with-profits life insurance products to ensure sufficiently high premiums
Finding the worst case scenario

- **Theorem:** $\bar{\mu}$ is a worst case scenario if $\forall t \in [t_0, n]$:

$$\bar{\mu}(t) = \arg\max_{(m_{jk})_{j \neq k} \in M(t)} \left\{ \sum_{j \neq k} \bar{\rho}_{j_0 j}(t_0, t) \left( b_{jk}(t) + \bar{V}_{k}(t) - \bar{V}_{j}(t) \right) m_{jk} \right\}$$

Thiele

$$\frac{\partial}{\partial t} \bar{V}_{j}(t) = r(t)\bar{V}_{j}(t) - b_{j}(t) - \sum_{k: k \neq j} \left( b_{jk}(t) + \bar{V}_{k}(t) - \bar{V}_{j}(t) \right) \bar{\mu}_{jk}(t) \text{ and } \bar{V}_{j}(n) = 0$$

Kolmogorov

$$\bar{\rho}_{j_0 j}(t_0, t_0) = 1_{(j_0 = j)} \text{ and } \frac{\partial}{\partial s} \bar{\rho}_{j_0 j}(t_0, s) = \sum_{l: l \neq j} \left( \bar{\rho}_{j_0 l}(t_0, s) \bar{\mu}_{lj}(s) - \bar{\rho}_{j_0 j}(t_0, s) \bar{\mu}_{jl}(s) \right)$$

- Numerically challenging due to initial condition at both $t_0$ and $n$
- General approach exists
  - Christiansen and Steffensen (2013a)
  - Christiansen, Henriksen, Schomacker and Steffensen
Cases where calculations simplifies

- Transition intensities independent
  \[ M(t) = \times_{\{(j,k) | j \neq k\}} M_{jk}(t) \]
  \[ \tilde{\mu}_{jk}(t) = \operatorname{argmax}_{m_{jk} \in M_{jk}(t)} \left\{ (b_{jk}(t) + \tilde{V}_k(t) - \tilde{V}_j(t)) m_{jk} \right\} \]
  Christiansen (2010)

- \( \operatorname{argmax} \) constant with respect to \( \tilde{p}_{j_0j}(t_0, t) \)
  Decouples Kolmogorov equations
  Christiansen and Steffensen (2013b)
Examples: Model

- Fixed standard Danish disability intensity
- Find worst case scenarios $\tilde{\mu}_{ad}$ and $\tilde{\mu}_{id}$
- Examples calculated using Actulus® Calculation Platform
Examples: Possible transition intensity scenarios

- Best estimate death intensity
  \( \mu^E(t) \) standard Danish intensity
- Scenarios based on Solvency II mortality and longevity stress
  \[ L(t) := (1 - 20\%) \mu^E(t) \]
  \[ U(t) := (1 + 15\%) \mu^E(t) \]

M(t): Independence

M(t): Dependence

M(t): Linear dependence
Independence

Upper $U(t)$ and lower $L(t)$ possible intensities

Worst case $\tilde{\mu}_{id}$

Worst case $\tilde{\mu}_{ad}$

Time (years)
Dependence

Upper $U(t)$ and lower $L(t)$ possible intensities

Worst case $\tilde{\mu}_{id}$

Worst case $\tilde{\mu}_{ad}$
Linear dependence

Upper $U(t)$ and lower $L(t)$ possible intensities

- Worst case $\tilde{\mu}_{ad} = \tilde{\mu}_{id}$
Conclusion

• Worst case scenarios related to VaR and Solvency II
• Find the worst case scenario by iteration
• Worst case scenarios can include interest rate
• Extends to portfolio of policies
References


