A COMPARISON OF
STOCHASTIC LOSS RESERVING METHODS

Ezgi Nevruz, Yasemin Gençtürk

Department of Actuarial Sciences
Hacettepe University
Ankara/TURKEY

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Outline

- Introduction
- Loss Process
- Simulation
- Loss Reserving Methods
- Application
- Conclusion
Our Aim

- Insurers should allocate an adequate reserve.
- Insurers need to select a suitable reserving method which estimates the expected liabilities as truly as possible.
- Profit of the companies does not only depend on the paid losses, but also the estimation of the future losses.

We aim to:

- estimate reserves using several loss reserving methods
- decide the suitable method for various scenarios by taking into account different performance criteria
Steps of a stochastic loss reserving process:

1. Defining the model structure for the loss
2. Preparing the loss data in accordance with the loss development triangle (upper-left triangle)
3. Obtaining the goal triangle (lower-right triangle) by means of the loss development triangle and suitable reserve estimation method
**Assumption:** Claims are settled at accident year or within the next \( n \) development years.

\[ S_{i,j} : \text{Incremental losses for accident year } i, \text{ development year } j \]

\[ L_{i,j} : \text{Cumulative losses for accident year } i, \text{ development year } j \]

\[
L_{i,j} = \sum_{k=1}^{j} S_{i,k}
\]

By assumption, incremental and cumulative losses are

- observable for calendar year \( i + j - 1 \leq n \)
- not observable for calendar year \( i + j - 1 \geq n + 1 \)
Loss development triangle

<table>
<thead>
<tr>
<th>Acc. Year (i)</th>
<th>Development Year (j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 ... k-1 k k+1 ... n-1 n</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
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<td>n-1</td>
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<td>n</td>
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</tbody>
</table>

- $S_{i,j}$
- $L_{i,j}$
- $i + j - 1 \leq n$
- $\hat{S}_{i,j}$
- $\hat{L}_{i,j}$
- $i + j - 1 \geq n + 1$

The time of a single claim

Occurrence Notification Loss Payments Re-Opening Loss Payment Closure

$t_1$ $t_2$ $t_3$ $t_4$ $t_5$ $t_6$ $t_7$ $t_8$ $t_9$
Algorithm:

Step 1: Generate claim numbers $N_i$ and individual claim amounts $\{C_{i,k}; k = 1, 2, \ldots, N_i\}$ for each accident year.

- Claim numbers: Poisson distribution
- Individual claim amounts: Pareto, Gamma and Lognormal distributions

Step 2: Obtain $\{U_{i,k}\}$ percentiles of each individual claim amounts $C_{i,k}$.

$$U_{i,k} = F(C_{i,k})$$
**Step 3**: For each $C_{i,k}; k = 1, 2, \ldots, N_i$ generate

- $X_{i,k,1}$ is the occurrence date
- $X_{i,k,2}$ is the reporting delay
- $X_{i,k,3}$ is the settlement delay

Here we define

$$r_{i,k} = \min\{\lfloor (X_{i,k,1} + X_{i,k,2}) \rfloor, n\}$$

$$R_{i,k} = \min\{\lfloor (X_{i,k,1} + X_{i,k,2} + X_{i,k,3}) \rfloor, n\}$$

Thus, the $k$th individual loss at the accident year $i$ is reported in $i + r_{i,k}$ calendar year and settled in $i + R_{i,k}$ calendar year (Narayan and Warthen, 2000).
**Step 4:** Obtain the developed loss amounts $\hat{C}_{i,k,j}$ increasingly with increasing $j$ for each individual loss amount by means of the inverse of the distribution function, i.e. $F^{-1}(U_{i,k})$:

If $C_{i,k}$ has lognormal distribution with log-scale parameter $\mu_j$ and shape parameter $\sigma_j^2$

$$\hat{C}_{i,k,j} = \begin{cases} 0 & ; j = 1, 2, \ldots, r_{i,k} \\ \exp\{\sqrt{2}\sigma_j[\text{erf}^{-1}(2U_{i,k} - 1) + \mu_j]\} & ; j = r_{i,k} + 1, \ldots, R_{i,k} \\ \exp\{\sqrt{2}\sigma_{R_{i,k}+1}[\text{erf}^{-1}(2U_{i,k} - 1) + \mu_{R_{i,k}+1}]\} & ; j = R_{i,k} + 1, \ldots, n \end{cases}$$

Here, $\text{erf}^{-1}$ is the inverse of the error function which can be shown as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$
Step 4: Continued

If $C_{i,k}$ has Pareto distribution with location parameter $\alpha_j$ and shape parameter $\theta_j$:

$$\hat{C}_{i,k,j} = \begin{cases} 
0 ; & j = 1, 2, \ldots, r_{i,k} \\
\theta_j \left[ \frac{1}{(1-U_{i,k})^{1/\alpha_j}} - 1 \right] ; & j = r_{i,k} + 1, \ldots, R_{i,k} \\
\theta_{R_{i,k}+1} \left[ \frac{1}{(1-U_{i,k})^{1/\alpha_{R_{i,k}+1}}} - 1 \right] ; & j = R_{i,k} + 1, \ldots, n 
\end{cases}$$
Simulation of a Loss Development Triangle (continued)

Step 4: Continued

If $C_{i,k}$ has Gamma distribution with scale parameter $\theta_j$ and shape parameter $\alpha_j$:

$$\hat{C}_{i,k,j} = \begin{cases} 
0 & ; j = 1, 2, \ldots, r_{i,k} \\
\theta_j P^{-1}(\alpha_j, U_{i,k}) & ; j = r_{i,k} + 1, \ldots, R_{i,k} \\
\theta_{R_{i,k}+1} P^{-1}(\alpha_{R_{i,k}+1}, U_{i,k}) & ; j = R_{i,k} + 1, \ldots, n
\end{cases}$$

Here, $P$ is the lower regularized Gamma function that

$$P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)} = \frac{\int_0^x t^{a-1}e^{-t} dt}{\Gamma(a)}$$
Step 5: Calculate the cumulative losses by the equation

\[ L_{i,j} = (1 + e)^{i-1} \sum_{k=1}^{N_i} \hat{C}_{i,k,j} \]

Here, because we obtain the cumulative losses increasingly, the incremental losses will be positive.
In the setting of reserves on the basis of information obtained from past years, one should be aware of the fact that inflation may have affected the values of claims.

1. The incremental losses $S_{i,j}$’s are calculated for $j = 2, 3, ..., n$ and each accident year.

2. The loss amounts are accumulated by accident and development year as

$$S_{i,j}(1 + e)^{n-i-j+1}$$

3. The inflation-adjusted cumulative losses $L_{i,j}$ are obtained from the inflation-adjusted incremental losses.
The development factor estimations $\hat{f}_j$ for $j = 2, 3, ..., n$ are obtained by

$$\hat{f}_j = \frac{\sum_{i=1}^{n-j+1} L_{i,j}}{\sum_{i=1}^{n-j+1} L_{i,j-1}}$$

Ultimate losses for each accident year are estimated as

$$\hat{L}_i = \hat{L}_{i,n} = L_{i,n-i+1} \prod_{j=n-i+2}^{n} \hat{f}_j$$

Finally, the reserve estimation for the $i$th accident year is calculated by the equation

$$\hat{R}_i = \hat{L}_i - L_{i,n-i+1}$$
Regression Methods

We are dealing with the loss development triangles that include positive incremental losses.

When the expected value of an incremental loss is $\theta_{i,j}$, we will obtain the unbiased estimate of $\theta_{i,j}$’s for $i = 1, 2, ..., n$ and $j = n - i + 2, ..., n$.

Under the assumption that the incremental losses are positive, the regression model is

$$Z_{i,j} = \ln(S_{i,j}) = \mu + \alpha_i + \beta_j + \varepsilon_{i,j}$$

where $\varepsilon_{i,j}$’s are iid $N(0, \sigma^2)$ distributed.
Under the assumption that the \( \{S_{i,j}\} \) r.v.s are independent and lognormally distributed, the \( \{Z_{i,j}\} \) r.v.s. are independent and normally distributed where \( i = 1, 2, \ldots, n; j = 1, 2, \ldots, n - i + 1 \).

\[
\mathbb{E}[Z_{i,j}] = X_{i,j} \beta, \quad \text{Var}(Z_{i,j}) = \sigma^2
\]

Therefore,

\[
\mathbb{E}[S_{i,j}] = \theta_{i,j} = \exp(X_{i,j} \beta + \frac{1}{2} \sigma^2)
\]

where \( X_{i,j} \) is the row vector of explanatory variables and \( \beta \) is a column vector of parameters.
Regression Methods (continued)
Model 1 & Model 2 & Model 3

Model 1:

\[ Z_{i,j} = \mu + \alpha_i + \beta_j + \varepsilon_{i,j} \Rightarrow \beta = [\mu, \alpha_2, ..., \alpha_n, \beta_2, ..., \beta_n] \]

Model 2:

\[ Z_{i,j} = \mu + (i - 1)\alpha + \beta_j + \varepsilon_{i,j} \Rightarrow \beta = [\mu, \alpha, \beta_2, ..., \beta_n] \]

Model 3:

\[ Z_{i,j} = \mu + (i - 1)\alpha + (j - 1)\beta + \gamma \ln(j) + \varepsilon_{i,j} \Rightarrow \beta = [\mu, \alpha, \beta, \gamma] \]

Here, there is a usual assumption that \( \alpha_1 = 0 \) and \( \beta_1 = 0 \) to make the model full rank (Verrall, 1991).

After \( \beta \)'s are estimated by the method of least squares, the unbiased estimations of \( \theta_{i,j} \)'s will be obtained for \( i = 2, ..., n \) and \( j = n - i + 2, ..., n \).
After \( \hat{\beta} = (X'X)^{-1}X'z \) is estimated, variance of the error is calculated as

\[
\hat{\sigma}^2 = \frac{1}{r-p}(z - X\hat{\beta})'(z - X\hat{\beta})
\]

where \( r = \frac{1}{2}n(n+1) \) is the number of observations, \( p \) is the number of parameters, \( X \) is the \((r \times p)\)-dimensional design matrix and \( z = [Z_{1,1}, Z_{1,2}, ..., Z_{1,n}, Z_{2,1}, ..., Z_{n,1}]' \) is the vector of observed losses.
Unbiased estimation of $\theta_{i,j}$ is obtained by

$$
\hat{\theta}_{i,j} = \exp(X_{i,j}\hat{\beta})g_m\left[\frac{1}{2}(1 - X_{i,j}(X'X)^{-1}X_{i,j}')s^2\right]
$$

where

- the biased estimate of $\sigma^2$ is $\hat{\sigma}^2$ and the unbiased estimate of $\sigma^2$ is $s^2 = \frac{r-p}{r-p} \hat{\sigma}^2$
- $m = r - p$ is the degree of freedom
- if the df of $\hat{\sigma}^2$ is $m$, then

$$
g_m(t) = \sum_{k=0}^{\infty} \frac{m^k(m+2k)}{m(m+2)...(m+2k)} \frac{t^k}{k!}
$$
\[ n = 11 \Rightarrow i = 1, 2, \ldots, 11; j = 1, 2, \ldots, 11 \]

- 10000 iterations

- Calculation of ultimate losses and actual reserves by the simulation of \((11 \times 11)\)-dimensional loss squares

- Estimation of reserves from the upper-left loss triangles

- Calculation of deviations and testing the performance of the reserving methods
Scenarios

System parameters:

- Inflation Rate (3 groups):
  - Low (6%) & Med (8%) & High (10%)

- Individual loss amount rv sample (4 groups)
  1. Low Mean (500), Low Variance (150²)
  2. Low Mean (500), High Variance (1000²)
  3. High Mean (5000), Low Variance (1500²)
  4. High Mean (5000), High Variance (10000²)

- Individual loss amount rv distribution (3 groups):
  - Pareto & Gamma & Lognormal

Thus, number of scenarios for each loss reserving method is $3 \times 4 \times 3 = 36$
### Scenarios (Summary)

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Inflation Rate</th>
<th>Distribution of Individual Loss Amount rv</th>
<th>Individual Loss Amount Sample</th>
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<td>Mean: %68</td>
<td>High: %10</td>
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<tr>
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<td>S18</td>
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### Results of the IACL Method

#### Table: Performance criteria of IACL

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<th>Sce</th>
<th>RMSE</th>
<th>MAPE</th>
<th>Sce</th>
<th>RMSE</th>
<th>MAPE</th>
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Results of the IACL Method (continued)

Mean Absolute Percentage Errors (Inflation-adjusted CL)

- MAPE values from 0.00507 to 1.98
- Scenarios from 'sce1' to 'sce36'

Ezgi Nevruz
# Results of the Regression Model 1

## Table: Performance criteria of Reg.Model 1

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<th>RMSE</th>
<th>MAPE</th>
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### Results of the Regression Model 2

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Results of the Regression Model 2 (continued)

Mean Absolute Percentage Errors (Regression Model 2)
## Results of the Regression Model 3

### Table: Performance criteria of Reg. Model 3

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</table>

Ezgi Nevruz

Application

02.04.2014

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Results of the Regression Model 3 (continued)

[Graph showing Mean Absolute Percentage Errors (Regression Model 3)]

Scenarios: s1, s2, s3, s4, s5, s6, s7, s8, s9, s10, s11, s12, s13, s14, s15, s16, s17, s18, s19, s20, s21, s22, s23, s24, s25, s26, s27, s28, s29, s30, s31, s32, s33, s34, s35, s36

MAPE values range from 0.00103 to 0.475.
Regression models give better results. They do not provide the best answers in all situations but they are consistent and give not only the point estimation but also a confidence interval.

Actuaries do not apply the CL method blindly. This method is efficient when the development factors are reasonable.

Inflation rate affects the performance of a loss reserving method.

We assume that the inflation rate is constant over the accounting period, however it could have been modelled by time series analysis.


Thank you.

http://www.aktuerya.hacettepe.edu.tr/pers/EzgiNevruz.php

ezginevruz@hacettepe.edu.tr