Stochastic Loss Reserving with Bayesian MCMC Models

Revised March 31

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April 2, 2014
The CAS Loss Reserve Database
Created by Meyers and Shi
With Permission of American NAIC

• Schedule P (Data from Parts 1-4) for several US Insurers
  – Private Passenger Auto
  – Commercial Auto
  – Workers’ Compensation
  – General Liability
  – Product Liability
  – Medical Malpractice (Claims Made)

• Available on CAS Website
  http://www.casact.org/research/index.cfm?fa=loss_reserves_data
## Illustrative Insurer – Incurred Losses

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Illustrative Insurer – Paid Losses

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Criteria for a “Good” Stochastic Loss Reserve Model

• Using the upper triangle “training” data, predict the distribution of the outcomes in the lower triangle
  – Can be observations from individual (AY, Lag) cells or sums of observations in different (AY,Lag) cells.
Criteria for a “Good” Stochastic Loss Reserve Model

• Using the predictive distributions, find the percentiles of the outcome data.

• The percentiles should be uniformly distributed.
  – Histograms
  – PP Plots and Kolmogorov-Smirnov Tests
    Plot Expected vs Predicted Percentiles
    KS 95% critical values = 19.2 for $n = 50$ and 9.6 for $n = 200$
Illustrative Tests of Uniformity
Data Used in Study

- List of insurers available from me.
- 50 Insurers from four lines of business
  - Commercial Auto
  - Personal Auto
  - Workers’ Compensation
  - Other Liability
- Criteria for Selection
  - All 10 years of data available
  - Stability of earned premium and net to direct premium ratio
- Both paid and incurred losses
Test of Mack Model on Incurred Data

Conclusion – The Mack model predicts tails that are too light.
Test of Mack Model on Paid Data

Conclusion – The Mack model is biased upward.
Test of Bootstrap ODP on Paid Data

Conclusion – The Bootstrap ODP model is biased upward.
Possible Responses to the Model Failures

• The “Black Swans” got us again!
  – We do the best we can in building our models, but the real world keeps throwing curve balls at us.
  – Every few years, the world gives us a unique “black swan” event.

• Build a better model.
  – Use a model, or data, that sees the “black swans.”
Proposed New Models are Bayesian MCMC

- Bayesian MCMC models generate arbitrarily large samples from a posterior distribution.
- See the limited attendance seminar tomorrow at 1pm.
Notation

- $w =$ Accident Year $w = 1,...,10$
- $d =$ Development Year $d = 1,...,10$
- $C_{w,d} =$ Cumulative (either incurred or paid) loss
- $I_{w,d} =$ Incremental paid loss $= C_{w,d} - C_{w-1,d}$
Bayesian MCMC Models

• Use R and JAGS (Just Another Gibbs Sampler) packages

• Get a sample of 10,000 parameter sets from the posterior distribution of the model

• Use the parameter sets to get \( \sum_{w=1}^{10} C_{wd} \), simulated outcomes

• Calculate summary statistics of the simulated outcomes
  – Mean
  – Standard Deviation
  – Percentile of Actual Outcome
The Correlated Chain Ladder (CCL) Model

- \( \logelr \sim \text{uniform}(-5,0) \)
- \( \alpha_w \sim \text{normal}(\log(\text{Premium}_w) + \logelr, \sqrt{10}) \)
- \( \beta_{10} = 0, \beta_d \sim \text{uniform}(-5,5), \text{for } d=1,\ldots,9 \)
- \( a_i \sim \text{uniform}(0,1) \)

\[
\sigma_d = \sum_{i=d}^{10} a_i \quad \text{Forces } \sigma_d \text{ to decrease as } d \text{ increases}
\]

- \( \mu_{1,d} = \alpha_1 + \beta_d \)
- \( C_{1,d} \sim \text{lognormal}(\mu_{1,d}, \sigma_d) \)
- \( \rho_d \sim \text{uniform}(-1,1) \)
- \( \mu_{w,d} = \alpha_w + \beta_d + \rho_d \cdot (\log(C_{w-1,d}) - \mu_{w-1,d}) \text{ for } w = 2,\ldots,10 \)
- \( C_{w,d} \sim \text{lognormal}(\mu_{w,d}, \sigma_d) \)
Predicting the Distribution of Outcomes

- Use JAGS software to produce a sample of 10,000 \( \{\alpha_w\}, \{\beta_d\}, \{\sigma_d\} \) and \( \{\rho\} \) from the posterior distribution.
- For each member of the sample
  - \( \mu_{1,10} = \alpha_1 + \beta_{10} \)
  - For \( w = 2 \) to 10
    - \( C_{w,10} \sim \text{lognormal} \left( \alpha_w + \beta_{10} + \rho_d \cdot (\log(C_{w-1,10}) - \mu_{w-1,10}) \right), \sigma_{10} \)
  - Calculate \( \sum_{w=1}^{10} C_{w,10} \)
- Calculate summary statistics, e.g. \( E \left[ \sum_{w=1}^{10} C_{w,10} \right] \) and \( \text{Var} \left[ \sum_{w=1}^{10} C_{w,10} \right] \)
- Calculate the percentile of the actual outcome by counting how many of the simulated outcomes are below the actual outcome.
The First 5 of 10,000 Samples on Illustrative Insurer with $\rho_d = \rho$

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Done in **R**

Done in **JAGS**
The Correlated Chain Ladder Model Predicts Distributions with Thicker Tails

- Mack uses point estimations of parameters.
- CCL uses Bayesian estimation to get a posterior distribution of parameters.
- Chain ladder applies factors to last fixed observation.
- CCL uses uncertain “level” parameters for each accident year.
- Mack assumes independence between accident years.
- CCL allows for correlation between accident years,
  \[
  \text{Corr}[\log(C_{w-1,d}),\log(C_{w,d})] = \rho_d
  \]
Examine Three Behaviors of $\rho_d$

1. $\rho_d = 0$ - Leveled Chain Ladder (LCL)

2. $\rho_d = \rho \sim \text{uniform} (-1,1)$ (CCL)

3. $\rho_d = r_0 \cdot \exp(r_1 \cdot (d-1))$ (CCL Variable $\rho$)
   - $r_0 \sim \text{uniform} (0,1)$
   - $r_1 \sim \text{uniform} (-\log(10) - r_0, -\log(r_0)/9)$
   - This makes $\rho_d$ monotonic $\in (0,1)$
Case 2 - Posterior Distribution of $\rho$ for Illustrative Insurer

$\rho$ is highly uncertain, but in general positive.
Generally Positive Posterior Means of $\rho$ for all Insurers

- **Commercial Auto**
- **Personal Auto**
- **Workers' Compensation**
- **Other Liability**
Case 3 - Posterior Distributions of \( r_0 \) and \( r_1 - \rho_d = r_0 \exp(r_1 \cdot (d-1)) \)

\[ \rho_d > 0 \]

Illustrative Insurer

\( \rho_d \) is generally monotonic decreasing
Generally Monotonic Decreasing $\rho_d$ for all Insurers

Commercial Auto

Personal Auto

Workers’ Compensation

Other Liability
## Results for the Illustrative Insured With Incurred Data

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Percentile: 86.03

CV: Coefficient of Variation
Results for the Illustrative Insured With Incurred Data

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Rank of Std Errors
Mack < LCL < CCL-VR ≈ CCL-CR
Compare SDs for All 200 Triangles
Test of Mack Model on Incurred Data

Conclusion – The Mack model predicts tails that are too light.
Test of CCL (LCL) Model on Incurred Data

$\rho_d = 0$

Conclusion – Predicted tails are too light
Test of CCL Model on Incurred Data

$\rho_d = \rho$

Conclusion – Plot is within KS Boundaries
Test of CCL Model on Incurred Data Variable $\rho_d$

Conclusion – Plot is within KS Boundaries
Improvement with Incurred Data

• Accomplished by “pumping up” the variance of Mack model.

What About Paid Data?

• Start by looking at CCL model on cumulative paid data.
Test of Bootstrap ODP on Paid Data

Conclusion – The Bootstrap ODP model is biased upward.
Test of CCL on Paid Data

Conclusion
Roughly the same performance as bootstrapping
How Do We Correct the Bias?

• Look at models with payment year trend.
  – Ben Zehnwirth has been championing these for years.

• Payment year trend does not make sense with cumulative data!
  – Settled claims are unaffected by trend.

• Recurring problem with incremental data – Negatives!
  – We need a skewed distribution that has support over the entire real line.
The Lognormal-Normal (ln-n) Mixture

\[ X \sim \text{Normal}(Z, \delta), \quad Z \sim \text{Lognormal}(\mu, \sigma) \]
The Correlated Incremental Trend (CIT) Model

- \( \mu_{w,d} = \alpha_w + \beta_d + \tau \cdot (w + d - 1) \)
- \( Z_{w,d} \sim \text{lognormal}(\mu_{w,d}, \sigma_d) \) subject to \( \sigma_1 < \sigma_2 < \ldots < \sigma_{10} \)
- \( I_{1,d} \sim \text{normal}(Z_{1,d}, \delta) \)
- \( I_{w,d} \sim \text{normal}(Z_{w,d} + \rho \cdot (I_{w-1,d} - Z_{w-1,d}) \cdot e^{\tau}, \delta) \)

- Estimate the distribution of \( \sum_{w=1}^{10} C_{w,10} \)

- “Sensible” priors
  - Needed to control \( \sigma_d \)
  - Interaction between \( \tau, \alpha_w \), and \( \beta_d \).
CIT Model for Illustrative Insurer

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Percentile

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Posterior Mean $\tau$ for All Insurers

**Commercial Auto**

**Personal Auto**

**Workers' Compensation**

**Other Liability**
Test of Bootstrap ODP on Paid Data

Conclusion – The Bootstrap ODP model is biased upward.
Test of CIT on Paid Data

- CA - CIT
  - KS D = 14.6
  - Crit. Val. = 19.2

- PA - CIT
  - KS D = 45.2 *
  - Crit. Val. = 19.2

- WC - CIT
  - KS D = 30.6 *
  - Crit. Val. = 19.2

- OL - CIT
  - KS D = 21.3 *
  - Crit. Val. = 19.2

- CA+PA+WC+OL
  - KS D = 25.3 *
  - Crit. Val. = 9.6

Better than when $\rho = 0$ – Comparable to Bootstrap ODP – Still Biased
Why Don’t Negative $\tau$s Fix the Bias Problem?

Low $\tau$ offset by Higher $\alpha + \beta$
The Changing Settlement Rate (CSR) Model

- \(\log(elr) \sim \text{uniform}(-5,0)\)
- \(\alpha_w \sim \text{normal}(\log(\text{Premium}_w) + \log(elr), \sqrt{10})\)
- \(\beta_{10} = 0, \beta_d \sim \text{uniform}(-5,5), \text{for } d = 1,\ldots,9\)
- \(a_i \sim \text{uniform}(0,1)\)

- \(\sigma_d = \sum_{i=d}^{10} a_i\) Forces \(\sigma_d\) to decrease as \(d\) increases

- \(\mu_{w,d} = \alpha_w + \beta_d \cdot (1 - \gamma)^{(w - 1)}\) \(\gamma \sim \text{Normal}(0,0.025)\)
- \(C_{w,d} \sim \text{lognormal}(\mu_{w,d}, \sigma_d)\)
The Effect of $\gamma$

- $\mu_{w,d} = \alpha_w + \beta_d \cdot (1 - \gamma)^{w-1}$
- $\beta$s are almost always negative! ($\beta_{10} = 0$)
- Positive $\gamma$ – Speeds up settlement
- Negative $\gamma$ – Slows down settlement
- Model assumes speed up/slow down occurs at a constant rate.
Distribution of Mean $\gamma$s

Predominantly Positive $\gamma$s
## CSR Model for Illustrative Insurer

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Test of CSR on Paid Data

Conclusion - Much better than CIT
Varying speedup rate???
Test of CIT on Paid Data

Better than when $\rho = 0$ – Comparable to Bootstrap ODP – Still Biased
Calendar Year Risk

Calendar Year Incurred Loss = Losses Paid In Calendar Year + Change in Outstanding Loss in Calendar Year

Important in One Year Time Horizon Risk
- 0 – Prior Year, 1 – Current Year
- $IP_1 = \text{Loss paid in current year}$
- $CP_t = \text{Cumulative loss paid through year } t$
- $R_t = \text{Total unpaid loss estimated at time } t$
- $U_t = \text{Ultimate loss estimated at time } t$

$$U_t = CP_t + R_t$$

**Calendar Year Incurred Loss**

$$IP_1 + R_1 - R_0 = CP_1 + R_1 - CP_0 - R_0 = U_1 - U_0$$

Ultimate at time 1 – Ultimate at time 0
Estimating the Distribution of The Calendar Year Risk

• Given the current triangle and estimate $U_0$
• Simulate the next calendar year losses
  – 10,000 times
• For each simulation, $j$, estimate $U_{1j}$
• CCL takes about a minute to run.

10,000 minutes????
A Faster Approximation

• For each simulation, $j$
  – Calculate $U_{0j}$ from $\{\alpha, \beta, \rho \text{ and } \sigma\}_j$ parameters
  – Simulate the next calendar year losses $CY_{1j}$

• Let $T =$ original triangle

• Then for each $i$
  – Calculate the likelihood
    $$L_{ij} = L(T,CY_{1i}|\{\alpha, \beta, \rho \text{ and } \sigma\}_j)$$
  – Set
    $$p_{ij} = \frac{L_{ij}}{\sum_j L_{ij}} \text{ and } \bar{U}_{1i} = \sum_j p_{ij} \cdot U_{0j}$$
A Faster Approximation

• \{\overline{U}_{1i} - U_0\} is a random sample of calendar year outcomes.

• Calculate various summary statistics
  – Mean and Standard Deviation
  – Percentile of Outcome (From CCL)
Illustrative Insurer
Constant $\rho$ Model

Figure 1 – 'Ultimate' Incurred Losses at $t=0$

- Mean = 39162
- Standard Deviation = 1906

Figure 2 – Next Calendar Year Incurred Losses at $t=0$

- Outcome = -54.62
- Percentile = 53.14

- Mean = -56
- Standard Deviation = 1248
Illustrative Insurer Variable $\rho$ Model

Figure 1 – 'Ultimate' Incurred Losses at $t=0$

Mean = 39091   Standard Deviation = 1906

Figure 2 – Next Calendar Year Incurred Losses at $t=0$

Outcome = -71.01
Percentile = 51.29

Mean = -90   Standard Deviation = 1295
Test of CCL Constant $\rho$ Model
Calendar Year Risk

CA - CY CCL
[Graph showing KS D = 17.2 and Crit. Val. = 19.2]

PA - CY CCL
[Graph showing KS D = 9.3 and Crit. Val. = 19.2]

WC - CY CCL
[Graph showing KS D = 18.4 and Crit. Val. = 19.2]

OL - CY CCL
[Graph showing KS D = 15.2 and Crit. Val. = 19.2]

CA+PA+WC+OL
[Histogram and graph showing KS D = 10.5 * and Crit. Val. = 9.6]

CA+PA+WC+OL
[Graph showing KS D = 10.5 * and Crit. Val. = 9.6]
Test of CCL Variable \( \rho \) Model
Calendar Year Risk

CA - CY CCL-VR

KS D = 15.7
Crit. Val. = 19.2

PA - CY CCL-VR

KS D = 9.4
Crit. Val. = 19.2

WC - CY CCL-VR

KS D = 15.1
Crit. Val. = 19.2

OL - CY CCL-VR

KS D = 15.9
Crit. Val. = 19.2

CA+PA+WC+OL

Frequency

KS D = 10.8 *
Crit. Val. = 9.6

CA+PA+WC+OL

Predicted

Expected
Short Term Conclusions

Incurred Loss Models

- Mack model prediction of variability is too low on our test data.
- CCL model correctly predicts variability at the 95% significance level.
- The feature of the CCL model that pushed it over the top was between accident year correlations.
- CCL models indicate that the between accident year correlation decreases with the development year, but models that allow for this decrease do not yield better predictions of variability.
Short Term Conclusions

Paid Loss Models

- Mack and Bootstrap ODP models are biased upward on our test data.
- Attempts to correct for this bias with Bayesian MCMC models that include a calendar year trend failed.
- Models that allow for changes in claim settlement rates work much better.
- *Claims adjusters have important information!*
Short Term Conclusions on Quantifying Calendar Year Risk

• Models with explicit predictive distributions provide a faster approximate way to predict the distribution of calendar year outcomes.

• Even though the original models accurately predicted variability, the variability predicted by the calendar year model was just outside the 95% significance level.
Long Term Recommendations

New Models Come and Go

• Transparency - Data and software released
• Large scale retrospective testing on real data
  – While individual loss reserving situations are unique, knowing how a model performs retrospectively should influence ones choice of models.
• Bayesian MCMC models hold great promise to advance Actuarial Science.
  – Illustrated by the above stochastic loss reserve models.
  – Allows for judgmental selection of priors.