

Trifurcation  
An Approach to Analyzing the Impact of Risk Treatment Alternatives

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**Abstract**

When companies engage in risk management activities, they can be seen as modifying their distributions of future outcomes. Often this takes the form of trading off a sure cost for a decrease in some potential risk of loss. Companies may carry out these activities with several different goals in mind, such as reduction in earnings volatility or preservation of capital. This paper will demonstrate a method for evaluating possible strategies by dividing the effects on costs and risks into three tranches (1) Earnings, (2) Impact and (3) Estate. The tranches can be associated with different interests of the various stakeholders. This trifurcation analysis can help to explain the risk reward trade-offs of different risk treatment alternatives and can aide in risk related decision making

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## Trifurcation<sup>3</sup>

Risk Management activities will pose a trade-off between one possible set of future outcomes and other less risky outcomes. There will usually be a cost for shifting to the less risky set of outcomes. Choosing between one set of outcomes and another is often difficult. It involves choosing between a sure cost and an increased potential for future losses. This paper will demonstrate a method for trifurcating (or dividing into three tranches) both the cost and the change in risk. The three tranches are (1) Earnings, (2) Impact and (3) Estate. This trifurcation analysis can help to explain the risk reward trade-offs of different risk treatment alternatives and can aide in risk related decision making.

Companies which engage in risk management can often view their choices in terms of the effect on the distribution of future outcomes. Any risk mitigation option can be considered by comparing the curve (say the cumulative distribution of net earnings) before and after applying the strategy. A successful strategy can be recognized by its effect – decreasing the likelihood of undesirable outcomes. At the same time there is often a cost for each strategy, for example in diminished expected profits. The question is then how to compare several different possible strategies. This is not a simple procedure – there is no standard widely accepted process. A common suggestion is to pick a risk measure and a return measure and to plot the results for each strategy, leading to the search for the “efficient frontier”.

We suggest that in some situations, management may have clear goals or constraints which are driving their search for risk reduction, and the Trifurcation approach seeks to evaluate each option in relation to those goals. We do this by dividing future scenarios into “tranches”, and identifying the tranche that corresponds to the management objective. Management would then favor strategies which benefit this tranche, sometimes at the expense of other tranches.

We will illustrate our approach with a series of examples in which a simple management constraint leads naturally to a division into three tranches, or “trifurcation”. In these examples we can associate the middle tranche with the primary objective of the risk management activity. Then the other two tranches can be more broadly defined as effects on volatility and estate that are outside of the primary objective.

The first example will be a stock fund where the manager is seeking to choose a risk mitigation alternative. The alternatives considered are all simple substitutions of money market short term fixed income investments for the stocks that previously made up the fund. The choices are examined from an expected return and volatility approach as well as from a Trifurcation approach.

Then a series of reinsurance examples are used to show how the Trifurcation approach displays the dynamics of a non-linear risk mitigation approach. Various choices of stop

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<sup>3</sup> The views expressed throughout the paper represent the opinions of the authors and do not necessarily coincide with the views and opinions of Willis Re.

loss reinsurance will be considered and compared using Trifurcation. The Trifurcation is shown to provide insights when used in conjunction with a Franchise Value calculation.

Finally, the Trifurcation method is used to highlight the difference between quota share reinsurance and the stop loss reinsurance where the earnings or the impact on risk are similar.

Through these examples, Trifurcation is shown to be a systematic method of isolating the degree to which a risk mitigation activity has impact upon the goals of the risk mitigation as compared to the impact that falls outside of those goals.

### **Definition of Trifurcation**

The Trifurcation method requires a set of random scenarios that are the simulated outcomes from a stochastic model of the system under study. The three tranches are defined in relation to a critical value and a mitigation strategy. They are constructed by comparing outcomes (say returns) before and after mitigation. The tranches are then defined as follows:

Earnings Tranche – corresponds to those scenarios where the critical value is not triggered before risk mitigation.

Impact Tranche –includes the scenarios where the critical value has been triggered before mitigation but not after. The Impact Tranche can also be more broadly defined to be the scenarios where the impact of the risk mitigation is seen to be beneficial to the risk management objectives of the firm.

Estate Tranche – corresponds to the scenarios where the critical value is triggered even after mitigation.

### **Example 1 – Allocation of Investment**

Let us imagine for this example that we are looking at the funds under management of a private equity fund and that the funds are the property of a family that says that they will move the money to a different manager if there is ever a 10% loss in the fund. The 10% loss is then clearly a critical value to the fund manager.

We assume the manager has generated a set of random scenarios that are the simulated outcomes from a stochastic model of the equity portfolio. The returns can be summarized by the following table (we assume the initial investment is \$500 M)

Table 1

Percentile	Return (\$M)	Return (%)
99.00%	279.20	55.8%
95.00%	207.61	41.5%
75.00%	117.25	23.5%
50.00%	54.09	10.8%
25.00%	(9.59)	-1.9%
20.00%	(24.33)	-4.9%
15.00%	(42.38)	-8.5%
10.00%	(66.88)	-13.4%
5.00%	(102.95)	-20.6%
1.00%	(186.04)	-37.2%
0.50%	(212.92)	-42.6%
0.40%	(224.79)	-45.0%
0.25%	(253.92)	-50.8%

From the table we see that the likelihood of a critical loss to the portfolio is 13.4%. We assume the manager considers that a 13% probability of job loss is somewhat excessive, and is looking for a risk mitigation alternative.

The manager considers the options of moving part of the funds into money market instruments. The expected returns on the money market instruments are about 1/10th the expected return from the equities, but the volatility is 1/200th as high. With an assumption of 100% correlation between money market and equity returns, she generates the following results for 5 possible allocations:

Table 2

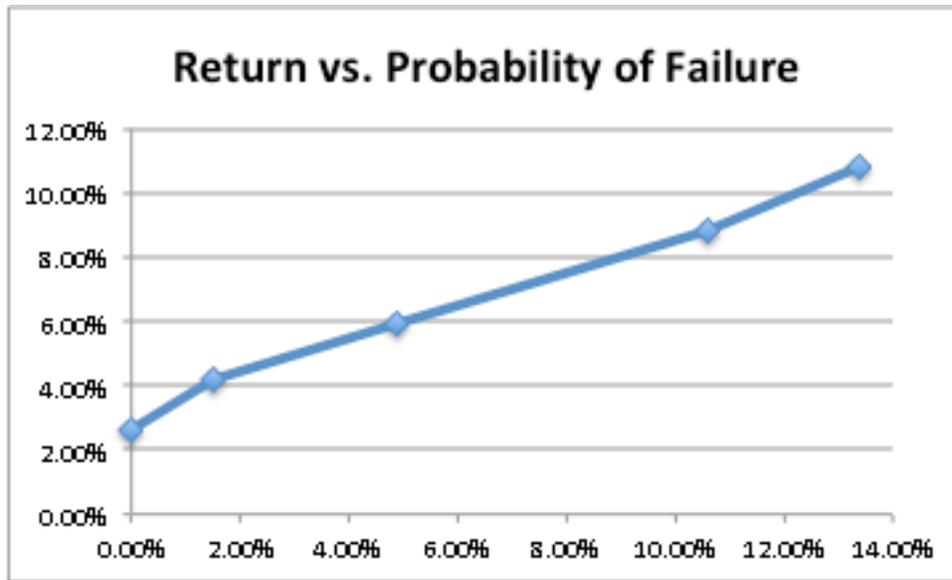
		Mean	Rate		PP
Stock	MM	Return	Of Return	CV	P(<-10%)
500	0	53.9	10.80%	1.82	13.4%
400	100	44.1	8.80%	1.79	10.6%
250	250	29.4	5.90%	1.68	4.9%
163	337	20.9	4.20%	1.55	1.5%
84	416	13.2	2.60%	1.28	0%

Chart 1



A standard Return vs. Risk plot shows the five alternatives. The plot simply graphically lays out the trade-offs with the standard deviation of returns as the risk measure. When viewed against the risk measure that is significant to of the fund manager, the plot would look more like the following Chart:

Chart 2



Notice here that the slope is sharply higher to go from \$84 million of stock to \$164 million than for the other steps. This means that the reward for taking on more risk is greater for that increase than for increases above that point.

Ultimately, the fund manager will want to know which of these options provides the “best” solution. The answer to that question is actually highly complex. Many approaches seek to boil the choices down to a single factor that is either higher or lower. In reality, however, there are always many considerations and an approach that sheds more light on more of those considerations might be desirable.

### **Trifurcation**

To carry out the trifurcation analysis, the risk manager separates the scenarios for each option as noted above:

The Impact Tranche is identified with those scenarios where the return for the stock only option (no mitigation) is a loss of more than 10%, but the return for the alternative option is a gain or a loss of at most 10%.

The Earnings Tranche is then defined as the scenarios where before risk mitigation (stocks only) there was a gain or the loss was less than 10%.

The Estate Tranche is the scenarios where the loss after mitigation is still greater than 10%.

Note that in each Tranche, the remaining scenarios are treated as unmitigated (in this case the stock only option).

For the asset manager, the trifurcation analysis would result in the following table:

Table 3

Stock	Change in Expected Value				
	<u>MM</u>	<u>Earnings Tranche</u>	<u>(Pos Earnings)</u>	<u>Impact Tranche</u>	<u>Estate Tranche</u>
500	0				
400	100	(12.62)	0.82	0.34	2.50
250	250	(31.54)	2.06	3.30	3.81
163	337	(42.52)	2.78	7.42	2.16
84	416	(52.48)	3.43	11.72	0.11

This analysis shows that for different choices of Money Market level, the relative effect on the Impact Tranche and the Earnings Tranche are quite significantly different. At 100 of MM, the change of the Estate Tranche is seven times larger than the change of the Impact Tranche. At 337 of MM, the Impact Tranche change is more than 300% the size of the change in the Estate Tranche, and more than 50% larger than the sum of the positive impact on the Earnings Tranche and the Estate Tranche ( $7.42 / \{2.16 + 2.78\}$ ) (note that the positive earnings column for the Earnings Tranche represents scenarios where the mitigation has a positive effect on the specific scenario, even though the overall expected effect of the mitigation for the tranche is negative).

Typically a risk mitigation activity will reduce the expected value of the Earnings Tranche and increase the value of the Impact Tranche. In some cases, the expected value of the Estate Tranche will increase as well.

We observe that in this case the critical value defines a point where the fund manager no longer has an interest in the outcome. Therefore any reduction of the expected value of the Earnings Tranche that “buys” an increase in the expected value of the Estate Tranche but not of the Impact Tranche could be considered “inefficient” in the eyes of the fund manager.

Thus a risk mitigation activity would be defined to be more efficient if more of the positive impact of the mitigation causes changes in the Impact Tranche. Positive impacts on the Earnings and Estate tranches would generally add unnecessarily to the cost of the mitigation. For example, as we showed above, moving only \$100M into the money market fund would be considered to be a highly inefficient option

In general if the critical loss amount is defined as an event that would bring the tenure of management to an end or even the independent existence of the company, then positive impacts on the Estate Tranche would inure to those with beneficial interests in the firm after than event. Current management may find risk mitigation impacts on the Estate Tranche to be an inefficient use of funds. Cost of a positive mitigation impact on this tranche may be high or low, due to the interplay of the size of the impact and its low likelihood. When the impact is extremely high, the price may be very high relative to

the likelihood especially when there is a direct counterparty whose own critical loss threshold might be affected.

Most commonly, there are mixed feelings about the impact of risk mitigation on the Earnings Tranche. The Earnings Tranche usually bears the burden of the cost of the risk mitigation. When there are positive effects on scenarios in the Earnings Tranche, the cost of that impact is generally high because of the relatively higher likelihood of the scenarios in the Earnings Tranche than of the other two tranches. Often, reduction of the range of possible severity of the Earnings Tranche is a secondary goal of the risk management efforts of the firm. To clarify discussions, the ideas of the Trifurcation approach may be beneficially applied to create more than one Earnings Tranche to clarify the costs and benefits there.

### Non-Linear Risk Mitigation Alternative

The insights of a Trifurcation analysis can be better seen when the alternatives have more complicated differences than linear combinations of the same two choices as in the example above.

For a non-linear example of risk mitigation, we consider an insurer under ratings pressure. Management has estimated that a loss of 15% of surplus would lead to a downgrade, and replacement of the current management team. To protect against this critical loss, management considers purchasing an aggregate stop loss cover. We assume that the company is writing at a 1 to 1 premium to surplus ratio, with an expense ratio of 20%. Thus a loss ratio of 95% (and a combined ratio of 115%) corresponds to the critical threshold (we are ignoring other effects on earnings such as investment income). The analysis compares different attachment options with a 10% loss ratio limit: For simplicity we assume the reinsurer expected loss ratio is the same for all options. Following the Trifurcation procedure as in the previous example leads to the following table (earnings in \$Millions):

Table 4 – 10% limit

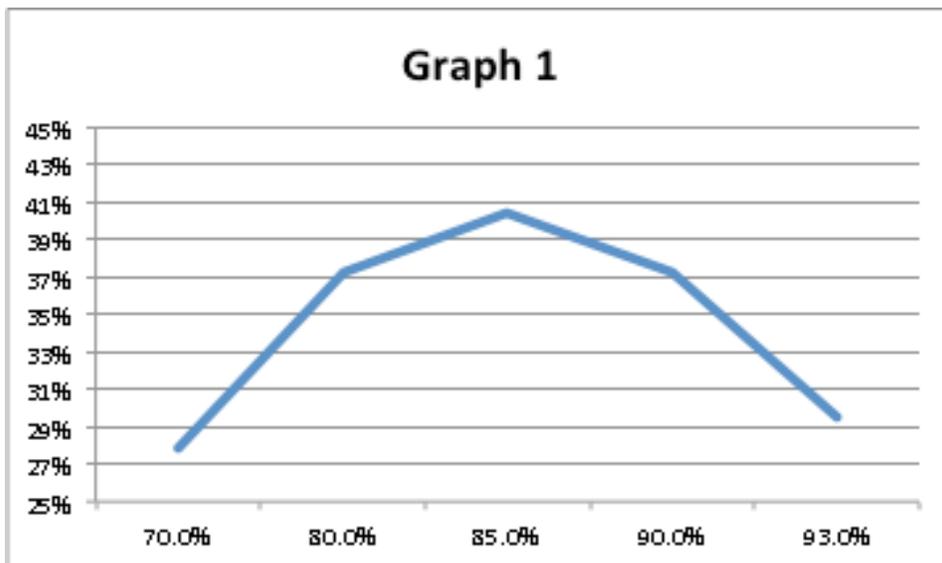
	Mean Ebit	CV Ebit	Prob Threshold Loss	Earnings	Impact	Estate
Gross	\$ 100.00	2.00	89.1%	-	-	-
stop loss attach 70.0%	\$ 84.92	1.93	92.1%	(20.52)	1.51	3.92
stop loss attach 80.0%	\$ 91.23	1.87	93.1%	(16.51)	2.88	4.85
stop loss attach 85.0%	\$ 93.50	1.87	93.5%	(15.07)	3.46	5.10
stop loss attach 90.0%	\$ 95.25	1.88	93.7%	(13.15)	3.13	5.27
stop loss attach 93.0%	\$ 96.09	1.89	93.9%	(11.48)	2.23	5.34

Table 4 shows that on a gross basis, the insurance produces a \$100M expected profit with a 10.9% probability of a loss over the 15% threshold. With an attachment at 70% of premium, the expected profits are reduced by \$15.08M which is the expected net cost of the reinsurance. For this type of risk mitigation, the premiums paid for the reinsurance

raise the likelihood of passing the threshold and the claims that are ceded to the reinsurer reduce the likelihood of passing the threshold. In the case of the 70% attachment, the likelihood of a loss in excess of threshold is reduced by 3%.

In each of the other cases, the higher attachment creates a lower premium AND a lower amount of ceded claims. The net cost decreases and the likelihood of a loss over the threshold decreases.

What is more interesting and perhaps more useful is the illustration of the expected values in the three tranches. The cost of the reinsurance for this reinsurance treaty hits the Earnings Tranche as it always will with the Trifurcation calculation. The benefits of the reinsurance treaty are split however, between the impact Tranche and the Estate Tranche. So if you assume that benefits that are paid in the Estate Tranche are less valuable, a measure of efficiency of the risk mitigation might be the proportion of benefits that are paid to the Impact Tranche.



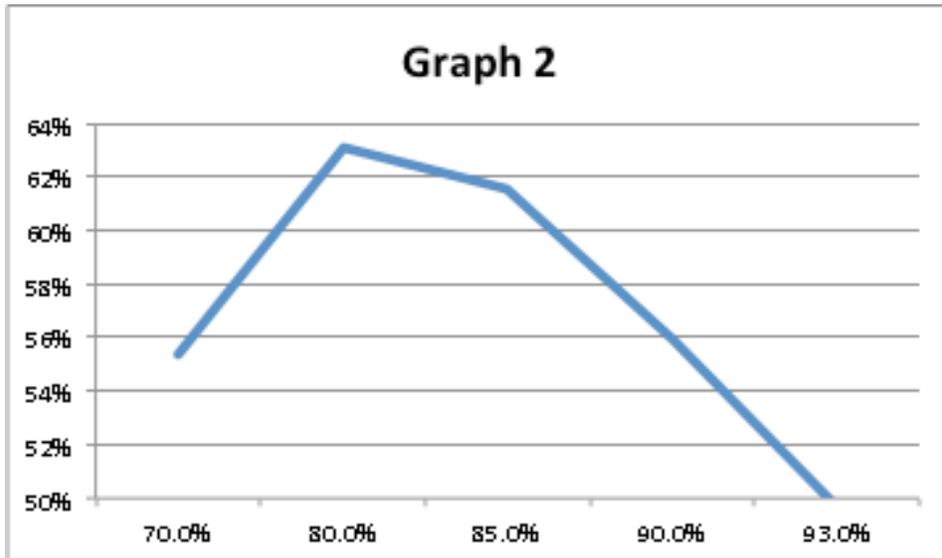
The plot of that ratio shows that the 85% attachment is the most efficient structure.

We can also consider these options with a 20% loss ratio limit:

Table 5 – 20% Limit

	Mean Ebit	CV Ebit	Prob Threshold Loss	Earnings	Impact	Estate
Gross	\$ 100.00	2.00	89.1%	-	-	-
stop loss attach 70.0%	\$ 76.15	1.80	95.1%	(37.02)	7.29	5.88
stop loss attach 80.0%	\$ 86.48	1.76	96.2%	(29.66)	10.18	5.95
stop loss attach 85.0%	\$ 90.06	1.78	96.5%	(25.26)	9.43	5.89
stop loss attach 90.0%	\$ 92.80	1.81	96.7%	(20.43)	7.40	5.82
stop loss attach 93.0%	\$ 94.09	1.83	96.8%	(17.41)	5.71	5.79

With the 20% limit, the net cost is of course much higher, but the likelihood of a loss exceeding Threshold is lower. The efficiency of the options is higher as well, since more of the effect of the reinsurance hits the Impact Tranche.



The 80% treaty is seen to be the most efficient with 63% of the benefits going to the Impact Tranche.

With the Trifurcation process, the idea of efficient risk mitigation can become more of a calculation and less of a vague discussion.

### Another View

In the foregoing illustration, the premium for the reinsurance is charged to each and every scenario, since in reality, it will be paid in all scenarios. However, it turns out that it is sometimes useful to look at the ceded losses separately from the reinsurance premiums.

Table 6 – 10% Limit

	<u>Expected (Ceded) Losses</u>			<u>RI Premium</u>		
	Earnings	Impact	Estate	Earnings	Impact	Estate
stop loss attach 70.0%	24.26	3.04	7.89	34.66	4.34	11.27
stop loss attach 80.0%	9.54	4.07	6.86	13.63	5.81	9.80
stop loss attach 85.0%	4.25	4.42	6.51	6.08	6.31	9.30
stop loss attach 90.0%	0.97	3.87	6.26	1.38	5.53	8.94
stop loss attach 93.0%	0.15	2.85	6.14	0.21	4.07	8.77

The expected (ceded) losses show how the reinsurance is expected to mitigate claims incurred by the insurer. In this illustration, the 90% attachment would be the point at which the Impact Tranche represents the highest percentage of total ceded losses. Treaties with the lower attachment are shown to pay a large amount of claims in the Earnings Tranche.

The RI Premium columns show an allocation of the single reinsurance premium between the three tranches. In all of the illustrated treaties the premium paid for the Impact Tranche benefits is actually less than half the total premiums.

Table 7 – 20% Limit

	<u>Expected (Ceded) Losses</u>			<u>RI Premium</u>		
	Earnings	Impact	Estate	Earnings	Impact	Estate
stop loss attach 70.0%	33.80	12.10	9.76	48.29	17.29	13.94
stop loss attach 80.0%	10.50	13.38	7.68	15.01	19.11	10.97
stop loss attach 85.0%	4.25	11.88	7.06	6.08	16.97	10.09
stop loss attach 90.0%	0.97	9.23	6.62	1.38	13.19	9.46
stop loss attach 93.0%	0.15	7.23	6.42	0.21	10.33	9.17

With the higher limit, the results shift and for the higher attachment points, the Impact Tranche dominates the results.

### Franchise Value

In none of the illustrations above were we selecting for the best reinsurance treaty, we were simply selecting for the most efficient. We neglected to consider that our choice might depress earnings more than another choice.

Panning (2006) and Bodoff (2011) both propose a Franchise Value calculation that brings all of the elements that are of concern to management of a firm into a single number. The earnings as well as the likelihood of experiencing a critical loss each have a major impact on the Franchise Value.

Table 8 – Franchise Value Added

	Value Added			
	Earnings	Impact	Estate	Total
10% Limit				
stop loss attach 70.0%	(184.48)	180.61	0.09	(3.78)
stop loss attach 80.0%	(74.70)	263.14	(45.76)	142.69
stop loss attach 85.0%	(33.68)	295.51	(59.92)	201.91
stop loss attach 90.0%	(7.68)	337.72	(69.82)	260.22
stop loss attach 93.0%	(1.21)	378.90	(74.51)	303.18
20% Limit				
stop loss attach 70.0%	(251.84)	306.61	(29.69)	25.08
stop loss attach 80.0%	(82.08)	417.63	(59.23)	276.32
stop loss attach 85.0%	(33.68)	517.28	(66.95)	416.64
stop loss attach 90.0%	(7.68)	653.03	(73.18)	572.17
stop loss attach 93.0%	(1.21)	751.44	(74.90)	675.33

With the Trifurcation approach to viewing the reinsurance transaction, it is shown that both the Earnings Tranche and the Estate Tranche have a negative impact on Franchise Value. In the case of the Earnings Tranche, the negative value impact is a result of the reinsurer margins. The value impact is the present value reflecting firm survivorship of the future margins with the assumption that the reinsurance is continued in each future year. For the Estate Tranche, the negative Franchise Value impact represents the entire premium paid. The premium is paid by the going concern company, but the reinsurance ceded losses paid are paid to the estate of the firm, after it has experienced a critical loss that throws it into receivership. In each case illustrated here, the reinsurance has a positive impact on the Impact Tranche because the positive value effect of the improvement in the likelihood of survival exceeds the reduction in value from the net cost of the reinsurance benefits.

### Comparison of Alternatives

So far the illustrations all show examples with very dissimilar outcomes. It is usually difficult to choose among those alternatives without bringing in additional constraints. Two possible constraints would be to focus on alternatives with similar income impact or on alternatives with similar impact on the probability of reaching the loss threshold critical value.

For this illustration, the linear type risk mitigation program represented by quota share reinsurance is compared to the non-linear stop loss. The comparison is made with a constraint that the reinsurance would (a) not reduce earnings by more than 10% and (b) would result in a probability of exceeding the critical loss threshold by no more than 7%. Most knowledgeable observers would be able to predict that the stop loss should be more efficient than the quota share.

Table 9 - Comparison

		Mean Ebit	CV Ebit	Prob Threshold Loss	Earnings	Impact	Estate
stop loss attach	77.7%	\$ 90.00	1.88	93.0%	(17.29)	2.59	4.70
Quota Share	20.0%	\$ 90.00	1.78	92.1%	(17.55)	1.37	6.18
Quota Share	25.0%	\$ 87.50	1.71	93.0%	(21.93)	2.24	7.19

The trifurcation calculation gives a repeatable numerical result that supports the conclusion that the stop loss is more efficient than the quota share. The stop loss provides 35% of its benefits to the Impact Tranche, while the two quota share examples provide 18% and 24% respectively to the Impact Tranche.

### Conclusion

The concept of Trifurcation allows the analyst to see the impact of a risk management action on different parties. The Earnings Tranche represents the interests of the shareholders in the ongoing concern before the mitigation activity. The Earnings Tranche will usually bear the cost of the risk mitigation activities. Most risk mitigation will reduce the expected value of the Earnings Tranche to create value in the Impact Tranche. The Impact Tranche represents the interests of those shareholders as they are enhanced by the risk mitigation activities. The Estate Tranche represents the interests of the policyholders and others who are concerned with the value of the firm after all of the shareholder funds have been exhausted. Most risk mitigation activities will create some benefit for the Estate Tranche in addition to the intended benefits for the Impact Tranche. The more efficient risk mitigations will create less such spillover into the Estate Tranche.

The risk taking capacity of the firm can often be represented by a critical loss value. The risk taking capacity is the amount of risk that can be tolerated without experiencing a critical loss and therefore ending the life of the firm.

Franchise Value can be tied directly to the Trifurcation analysis to see these dynamics as the play out for different reinsurance alternatives.

If Franchise Value is not a viable metric for a firm, then the choices of most efficient risk mitigation can be used. Usually the choices will need to be subject to earnings and/or critical loss probability constraints. Then treaties with similar expected earnings and/or critical loss probabilities can be compared. The examples given here illustrate the insight from Trifurcation under either constraint.

## **References**

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Neil M. Bodoff, FCAS, MAAA, *Sustainability of Earnings: A Framework for Quantitative Modeling of Strategy, Risk, and Value* (2011)