The session will begin with a dilemma that confronts actuaries when relying upon a single model to measure the variability around a central estimate based on multiple models. We will then provide an overview of the basic building blocks to estimating reserve variability and will then address a component of reserve variability that is often overlooked: model uncertainty. This session will present practical methodologies for incorporating model uncertainty into the actuary’s estimate of uncertainty and will use a case study to demonstrate their use.

Dilemma
Dilemma

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:

\[
\text{Model A point estimate}
\]
\[
\text{Model B point estimate}
\]

- How do we estimate the uncertainty in our central estimate?

Dilemma

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:

\[
\text{Central Estimate} = \frac{(\text{Model A} + \text{Model B})}{2}
\]

- Assume the actuary selects the central estimate to be the average of the point estimates from the two models:

\[
\text{Selected point estimate}
\]

- How do we estimate the uncertainty in our central estimate?

Dilemma

- One way might be to estimate uncertainty using one of our underlying models as the basis:
  - Using Model B as the basis for estimating uncertainty:
  - This raises two issues:
  1. Central estimate (red) is not "central" within distribution
  2. Model A point (blue) estimate appears unlikely yet given 50% weight.
Dilemma

- The first issue can be resolved by scaling:
  
  - **Additive Scaling:** \( y' = y + (central \ estimate - y) \)
  
  - **Multiplicative Scaling:** \( y' = \frac{y \times (central \ estimate)}{y} \)

- The central estimate (red) is now “central” with distribution
- However, the second issue remains:
  - Model A point estimate (blue) still appears unlikely yet given 50% weight

Incorporating Model Uncertainty

Overview of Approach
Uncertainty in an Actuarial Central Estimate

- Measuring uncertainty is a challenge in our profession because the unpaid claim process is unknown and the output from this process is not a repeatable exercise.
- Many approaches exist to estimating the uncertainty in an unpaid claim estimate:
  - Mack, Bootstrapping, MCMC, practical stochastic simulation
- Two common themes in these approaches:
  - Prediction error is comprised of parameter error and process error.
  - A single model is assumed to be representative of the unpaid claim process.

Two common themes in these approaches:
- Prediction error is comprised of parameter error and process error.
- A single model is assumed to be representative of the unpaid claim process.

However, this is rarely the case, and actuaries will commonly employ multiple models.
We therefore need some way in which to reflect the additional implied uncertainty among the models.

Our Approach

Generate a distribution comprised of simulations about each model using current approaches:
- Bootstrapping, simulation from an assumed distribution, simulation from analytical models, simulating and scaling, etc.

Weighted sample
Our Approach

- Generate a distribution comprised of simulations about each model using current approaches:
  - Bootstrapping, simulation from an assumed distribution, simulation from analytical models, simulating and scaling, etc.
- Weighted sample
- Aggregating results across multiple years requires additional rigor:
  - Rank Tying and Model Tying approaches are available to generate aggregate distributions

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Weighted Sampling

- Single Years

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Sampling of methods

- Start by creating simulated distributions for each of Model A and B:
Sampling of methods

- Create a ‘Model Matrix’ based on selected weighting
- In this case, we will use 50-50 weighting between Model A and B
- Simulations are pulled from each model based on this ‘Model Matrix’

Comparison of results from Weighted Sampling between Model A and Model B:

- Using weighted sampling
- Distribution around Model B
- Distribution around Model A
- Combined distribution using weighted sampling

Comparison of results from Weighted Sampling vs. Scaling:

- Using weighted sampling
- Using scaling

Using weighted sampling vs. Using scaling
Sampling of methods

- Adjusting our underlying weights will shift the resulting distribution accordingly:

![Diagram](image1.png)

Sampling of methods Multi-modal distributions

- Weighted sampling may produce 'lumpy', or multi-modal, probability density distributions.
- However, the probabilities across a range of outcomes may be more easily interpreted using the associated cumulative probabilities graph.
- Further adjustments could be made to the simulated results if such an outcome was deemed problematic.

![Diagram](image2.png)

Sampling of methods

- So far, we have considered a scenario with just a single set of simulations.
- What if we have multiple sets of predictions?
  - Multiple accident years, for example.

![Diagram](image3.png)
Weighted Sampling

Multiple Years

Again, for each time period, we can create a 'Model Matrix' based on the selected weighting.

Sampling of methods Multiple Year Aggregations

A note on simulation tying

- Typically, the methods that are used to generate the simulations around each of the underlying models do not treat each accident year in isolation, but rather produce year-by-year results that are intrinsically related to each other.
- This is reflected in each and every simulation, which we can think of as 'strings'.
- This means that we are able to calculate the total unpaid amount for each simulation by simply summing across each row.
- In this manner, any accident year correlation that is inherent to the model can be maintained.
Sampling of methods  Multiple Year Aggregations

A note on simulation tying: Another Dilemma

- What happens when we mix samples from different models in simulation 'strings'?
- Where a break occurs in a 'string', we destroy any correlation that may have been included in our model.

If we simply randomly-arrange our samples across simulations, we essentially destroy any year-by-year correlation in our results and we are no longer able to sum across the rows to get the total (unless this is desired).

Going back to our sampled simulations - because we sampled independently for each time period, we have broken the links intrinsic to the underlying model(s).
With this example (equal weighting for each accident period), we can get around the problem by sampling just one time and ensuring that we pick the same simulation for every time period.

This approach achieves the objective in that each individual accident period reflects the desired weighting and we maintain the correlation inherent to each individual simulation. However...

...what if our selected weightings vary for each origin year? In this case, we need to sample independently to maintain appropriate year-by-year representation...

We require some manner of rearranging our simulations to reflect underlying correlations. Going back to our earlier example, sampling individually by years, we suggest 2 ways in which to achieve this...
Sampling of methods Multiple Year Aggregations

1) Rank Tying:
Rearrange the sampled simulations themselves using a 'borrowed' correlation matrix

2) Model Tying:
Rearranging the Weighted Samples 'Model Matrix' prior to pulling through the reserves from the underlying model

Aggregating Results
Rank Tying
Aggregating Results  

**Rank Tying**

- This approach involves rearranging the sampled, simulated reserves

- We can ‘borrow’ a correlation matrix from one of the underlying models
- We do this by calculating the reserve ranks for each year for the underlying models

- We then select which model to use as the basis for our rank-tying (in this case, Model B)...
- ...and reorder the sampled simulations accordingly on a year-by-year basis
Aggregating Results  

**Rank Tying**

- Once rearranged, we can then sum across the rows to calculate a total reserve for each simulation.

- **Aggregating Results**  
  **Rank Tying: Summary**

- **Rank Tying** is a means of combining simulations across origin periods while maintaining the same parameter variance dependency structure associated with one of the underlying projection models.
- In essence, this approach assumes that the introduction of model uncertainty does not produce any dependency across origin periods.
- Rank Tying dependencies across accident years:
  - Process Error = None
  - Parameter Error = Select a single model for source
  - Model Error = None
- Should there be correlation among accident years for model uncertainty?
  - It may be argued that if a model is biased to overestimate or underestimate then it will likely have a similar bias across all origin periods.

- **Aggregating Results**  
  **Model Tying**
Aggregating Results Model Tying

- This method also involves reordering the simulations.
- However, in this case, we will be rearranging at the 'Model Matrix' stage, prior to pulling through the reserves from the underlying model.

Sampling error may mean that we do not achieve an exact 50/50 split in each year so 'perfect strings' are not always possible.

We wish to reorder the 'Model Matrix' to maximize the degree to which 'A's in one year are grouped with 'A's in other years, and the degree to which 'B's are grouped with 'B's.

We do this to maximize the correlation of the method selected in each of the accident years.

We can now select the samples from our underlying methods using the sampling 'Model Matrix' reordered such that we maximize the 'model correlation'…
Aggregating Results Model Tying

...allowing us to simply sum across the sampled simulation ‘strings’ to derive our set of total simulated reserves.

Aggregating Results Model Tying: Summary

- Using the Method Tying approach ensures that, where possible, the original ‘strings’ of simulations through each year are kept intact, thereby inherently including the dependencies implied by the underlying models.
- However, where perfect ‘string’s aren’t possible due to changing weights, we are essentially breaking origin period correlation caused by parameter error within a model, as we are combining simulations from different models randomly.
  - This may be a desirable effect:
    - Pre-sorting the original sets of simulations (prior to sampling) imposes a proxy dependency between models.
    - Rank Tying dependencies across accident years:
      - Process Error = None
      - Parameter Error = Yes, to the extent selection weights between models implies it should exist
      - Model Error = Yes
Aggregating Results Model Tying

- In a situation where equal weights are applied to each accident year, this approach will yield very similar results to the method suggested earlier – i.e. sampling just once and ensuring that the same simulation is picked for each time period:

- Weighted sampling at individual years, then Model Tying
- Weighted sampling at total

Aggregating Results Summary

- We have outlined three ways in which yearly reserve uncertainty estimates can be aggregated to determine the variability around the total (i.e. all year) unpaid loss estimates:
  - Weighted sampling at a total level
  - Weighted sampling and re-arranging sampled simulations with Rank Tying
  - Weighted sampling and re-arranging the Model Matrix with Model Tying

- It is not always easy to predict how the approaches will compare as it depends on the weightings employed and the results of the respective models across accident years.
Aggregating Results Summary

- All three approaches are scalable to allow for the incorporation of multiple models and multiple accident years in the estimate of reserve uncertainty.
- Furthermore, the Rank Tying and Method Tying approaches involve sampling at the individual year level and therefore also support the ability to apply weights specific to each accident year.
- This allows actuaries to reflect the same weighting philosophy in their uncertainty estimate as employed in their selection of the central estimate.

Case Study

Application of Approach

Case Study Underlying Models

- Three models are investigated.
- For the central estimate, each model is given equal weight (for each accident year).
Case Study Variability around individual models

- Three models are investigated
- For the central estimate, each model is given equal weight (for each accident year)
- Traditional methods are used to produce predictive distribution around each model (based on Bootstrap approach)

Case Study Variability around multiple models

- We are now faced with the challenge of deriving an estimate of the uncertainty around our prediction, reflecting each model used
- We can employ alternative methods for deriving the uncertainty for individual accident years:
  - Model scaling (using model B)
  - Weighted sampling (using weights)

Case Study Aggregating the results

- Finally, we must aggregate the individual accident year results to calculate the total variability estimate
- With the Model Scaling approach, the total estimate is relatively easy to derive as we are utilizing the simulation strings from a single underlying model
Case Study Aggregating the results

- Similarly, we can utilize the Rank Tying and Model Tying approaches to derive the total variability estimate for our weighted samples.

Incorporating Model Error into Actuary’s Estimate of Uncertainty

Summary

- The uncertainty of a prediction is comprised of three components:
  - A number of commonly-employed approaches compute uncertainty under the assumption that a single model is representative of the phenomenon
  - Model error is evident when the actuary places reliance on multiple models as being instructive of their central estimate of unpaid amounts
  - Weighted sampling is an approach that can be used to incorporate model uncertainty around a central prediction
  - Rank Tying and Model Tying are practical approaches that can be used to incorporate model uncertainty into an aggregation of multiple predictions (e.g. multiple accident years)
  - What we produce is a predictive distribution (or a range around our predictions)
  - Such approaches allow the actuary to tackle their analysis of uncertainty in an intuitively similar manner to how they derive their central estimate – i.e. with the use of multiple models and application of weights